A Joint Optimization Framework for Request Scheduling and Energy Storage Management in a Data Center

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***Abstract*—This paper addresses the problem of profit maximization for a data center with battery banks deployed at various levels of the power hierarchy. An optimization framework that covers the request dispatch, server resource allocation, and battery charging management is proposed. Instead of controlling the input/output power of the batteries after knowing the power profile of all other components of the data center as in a set of prior work, an optimal management policy is proposed which adjusts the power consumption (or supply) of servers and the battery banks at the same time. A response time dependent revenue model is adopted based on the delay estimation using the generalized processor sharing model. The rate capacity effect and the state of health degradation of the batteries, as well as the conversion and transmission loss in the power delivery network, are considered for the purpose of accurate power modeling and utility cost estimation. It is shown that the problem can be transformed into a series of convex optimization problems and then solved using standard solvers.**

I. Introduction

Since the concept of cloud computing [1] is brought up, it has been continuously drawing attention because of its major advantages in on-demand self-service, ubiquitous network access, location independent resource pooling, and transference of risk [2]. To host the cloud computing service for the ever-growing client population, warehouse scale data centers are built up. According to [3], electricity used by the data centers has grown to account for 3% of the overall consumption in the US. Data centers are power-hungry (each usually consumes megawatts of electricity power) and require careful power provisioning to ensure that the power infrastructure is utilized efficiently [4]. According to the information provided in [5], [6], the utility bill (mainly electricity) can contribute up to 24% to the total amortized monthly cost of a 10MW data center. Consequently, controlling the expenditure on the required electricity energy becomes a crucial part for data center operators in order to maximize the net profit.

The most direct approach to control the utility cost of a data center is to reduce its total power consumption without any significant performance loss. With the flexibility provided by power management techniques, some prior work has focused on finding the desirable tradeoff between the processing latency and total power consumption. For instance, authors of [7] study the resource allocation problem in a multi-tier cloud computing system aimed at profit maximization, while authors of [8] propose an algorithm to optimally dispatch the service requests and set the dynamic voltage and frequency scaling (DVFS) policy for each processing core. Since only constant utility pricing is considered in these references, the achieved improvement is limited.

With the emerging practice of dynamic energy pricing [9], utility companies vary the energy price based on the time-of-day or other factors to balance the supply and demand, thus reducing their capital expenditure. The topic of *peak power shaving* in power management has also become of importance for data center operators because the utility cost not only depends on *how much* energy is consumed but also *when* the energy is consumed. Amongst a variety of peak power shaving techniques including DVFS [10], virtual-machine-aware power budgeting [11], and online job migration [12], one of the most common approaches is to extend the usage the energy storage devices, e.g. batteries, supercapacitors, etc., in a data center. By provisioning larger energy storage capacity, the energy storage devices can not only serve as uninterruptible power supply (UPS) units but also help achieve peak power shaving because of their capability of partially migrating the power demand between different hours of a day.

When trying to design effective power management techniques involving batteries or other types of energy storage devices at different levels of a data center (e.g. cluster, power distribution unit, rack, server), one must carefully select an appropriate model for these devices. Admittedly, some accurate battery models, either based on electrochemical analysis [13] or in the form of equivalent electrical circuits [14], [15] are difficult to be incorporated into an optimization problem with efficient solutions. Nevertheless, the adopted battery model should not be over simplified in order to avoid a significant loss of accuracy, which may even lead to misleading results. Therefore, some key phenomena should be captured when modeling the battery charging/discharging process, e.g. *rate capacity effect* and the *state of health (SoH)* *degradation*. Unfortunately, these phenomena are partially or completely overlooked in prior work that uses energy storage devices for peak power shaving in a data center such as [16]–[18].

The rate capacity effect, which can be captured by Peukert's law [19], suggests that the decreasing rate of the remaining capacity in a battery cell or, equivalently, that of the remaining energy is actually a super-linear function of the discharging current. In other words, the maximum amount of energy that can be drawn from a battery cell in one cycle decreases with higher discharging currents. As suggested by the authors of [20], if the rate capacity effect is overlooked, large discharging peaks are likely to appear to allow for maximum amount of peak shaving. As a result, the predicted improvement of some battery management techniques will be too optimistic compared to the measurement due to overestimation of the available energy storage.

In addition to the dependence on the magnitude of the discharging current, the effective energy capacity of a fully charged battery cell may vary after each discharging/recharging cycle, which is defined as the SoH degradation. The SoH degradation suggests that the maximum capacity of a battery cell (when it is fully charged) will decrease after each discharging/recharging cycle from its nominal capacity gradually towards a threshold value. The decreasing rate is determined by various factors including the average state of charge (SoC) during the discharging process and the swing of the SoC (i.e. the difference between the starting point and the ending point of the discharging process). After its maximum capacity reaches the threshold value, the battery cell should be replaced. Although some prior work identifies the problem of SoH degradation, the SoH model is simplified as a function of depth of discharge (DoD) and the effect of the average SoC during the discharging process is overlooked.

In this paper, we take into consideration various aspects in data center management including the request dispatch, resource allocation, and peak power shaving using the battery banks at different levels of the power hierarchy of a data center to minimize the overall cost from the perspective of a data center operator. Rather than first design management policies for each part separately and then combine them together, we propose a joint optimization framework that finds the request flow control and the battery management policies at the same time. By doing so, we can perfectly address the inter-dependency between these aspects. A generalized time-of-use (TOU) pricing model that allows energy sell back is considered. The average response time of each request is calculated using the generalized processor sharing (GPS) model. Batteries are installed as the energy storage devices at each level of the power hierarchy of the data center. Both the rate capacity effect and the SoH degradation of the batteries are accounted for to realistically model the energy conversion efficiency of the battery.

The rest of this paper is organized as follows. Section II introduces the system model we will use. The optimization problem of cost minimization is formulated and solved using standard convex optimization techniques in Section III. Section IV presents the experimental results. And the last section is the conclusion.

II. SYSTEM MODEL

A. Data center structure

In this paper, we consider the data center structure comprised of J heterogeneous servers labeled from 1 to J and a set of battery banks (as shown in Fig. 1[1]). The power infrastructure hierarchy has a tree structure with the root node directly connected to the power grid and each leaf node directly connected to a server, and each node is equipped with a battery array. We index each battery array by a label (k, m), where k is the level it is at and m is its index within the level. Therefore, if there are K levels in total and the number of nodes at the k-th level is denoted by M_k, the battery array at the data center level is labeled by $(1, 1)$, and the server level battery arrays are labeled by $(K, 1)$ – (K, J). Without loss of generality, we assume that the battery array (K, j) is connected to the same node as the j-th server. The nominal charge capacity (in the unit of A·h) of battery array (k, m) is denoted by $C_{k,m}^{nom}$. All battery arrays can exchange energy with the power grid through power lines but may suffer from energy loss in transmission. The transmission efficiency between two adjacent levels k and $k+1$ is denoted by $\eta_{T,k}$. Furthermore, we define a function $S(k, m)$ where $S(k, m) = m'$ means that the parent node of the node to which battery array (k, m) is connected is $(k-1, m')$. In the case that battery arrays are only installed on a portion of the nodes, one can simply set $C_{k,m}^{nom}$ to 0 for the nodes without energy storage capability.

B. Response time modeling

The GPS model is used in this paper to calculate the average response time for each request. In order to find the analytical form of the response time, we assume that both the arrival and the processing of each request follow Poisson processes. To consider different amount of workload and utility prices at different time of a day, we partition each day into L time periods, and each time period $l \in \{1, 2, \ldots, L\}$ has a duration of T_l. If the average arrival rate of the service requests in time period l is denoted by λ_l, and the probabilities that a service request is dispatched to a specific server in time period l are denoted by $p_{1,l}, \ldots, p_{J,l}$, then the request arrival rate for the j-th server in time period l can be calculated as $p_{j,l} \cdot \lambda_l$. If the maximum processing rate of the j-th server is denoted by μ_j, and the proportion of

[1]The DC/AC, AC/DC, and DC/DC converters in the infrastructure are omitted in the figure, but the power conversion efficiency is accounted for in the proposed model.

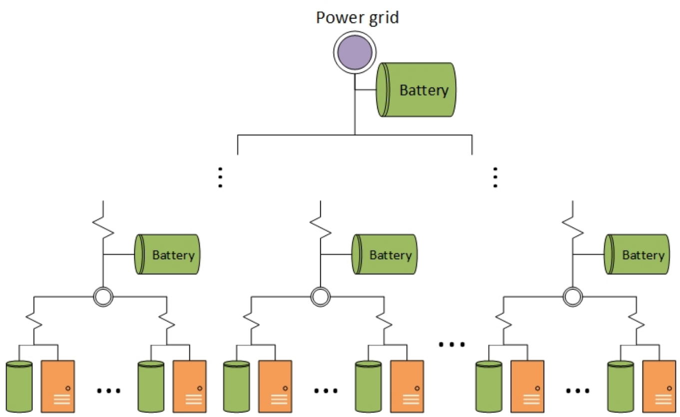

Figure 1. The structure of a data center with multi-level power hierarchy and energy storage devices

resources that is allocated to process the incoming requests by the j-th server in time period l is denoted by $\phi_{j,l}$, then the average response time for a service request dispatched to the j-th server, denoted by $R_{j,l}$ can be calculated as

$$R_{j,l} = \begin{cases} \dfrac{1}{\phi_{j,l} \cdot \mu_j - p_{j,l} \cdot \lambda_l}, & p_{j,l} > 0 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

And the average response time for all the requests in time period l, denoted by $\bar{R}_l$, can be calculated as

$$\bar{R}_l = \sum_j p_{j,l} R_{j,l} \tag{2}$$

Please note that as long as $R_{j,l}$'s are non-negative, $\bar{R}_l$ is a convex function of $p_{j,l}$'s, and it is also a convex function of $\phi_{j,l}$'s.

C. Battery modeling accounting for the rate capacity effect and the SoH degradation

Because of the rate capacity effect, the equivalent energy increasing/decreasing rate of the battery is different from the power consumed/provided by the battery. If we denote the input power of the battery as P_{bat}, and the equivalent energy increasing rate of the battery by $P_{bat,eq}$, then the relationship between P_{bat} and $P_{bat,eq}$ can be expressed as

$$P_{bat} = \begin{cases} \dfrac{1}{\eta_C} V_{bat} I_{ref} \left(\dfrac{P_{bat,eq}}{V_{bat} I_{ref}} \right)^{\gamma_C}, & \dfrac{P_{bat,eq}}{V_{bat} I_{ref}} > 1 \\ \dfrac{1}{\eta_C} P_{bat,eq}, & 0 \leqslant \dfrac{P_{bat,eq}}{V_{bat} I_{ref}} \leqslant 1 \\ \eta_D P_{bat,eq}, & -1 \leqslant \dfrac{P_{bat,eq}}{V_{bat} I_{ref}} \leqslant 0 \\ -\eta_D V_{bat} I_{ref} \left(\dfrac{|P_{bat,eq}|}{V_{bat} I_{ref}} \right)^{\frac{1}{\gamma_D}}, & \dfrac{P_{bat,eq}}{V_{bat} I_{ref}} < -1 \end{cases} \tag{3}$$

where γ_C and γ_D are Peukert factors for charging and discharging, respectively, which have the typical values ranging from 1.1 to 1.3 depending on the battery type, η_C and η_D are the power conversion efficiency for charging and discharging process, respectively, V_{bat} is the storage terminal voltage which is near-constant, and I_{ref} is the reference discharging current with negligible rate capacity effect which is proportional to the nominal capacity of the battery. When $P_{bat,eq} > 0$, Eqn. (3) corresponds to the charging process, and when $P_{bat,eq} < 0$, Eqn. (3) corresponds to the discharging process. Please note that since both γ_C and γ_D are greater than 1, and both η_C and η_D are less than 1, it can be proved that P_{bat} is a convex increasing function of $P_{bat,eq}$.

The SoH of the battery is defined as

$$SoH = \frac{C_{bat}^{full}}{C_{bat}^{nom}} \tag{4}$$

where C_{bat}^{nom} is the nominal charge capacity of the battery when it is new, and C_{bat}^{full} is the degraded charge capacity of the used battery. With SoH degradation, the SoC of the battery should be defined as

$$SoC = \frac{C_{bat}}{C_{bat}^{full}} \tag{5}$$

where C_{bat} is the remaining charge in the battery.

We calculate the SoH degradation of each charging/discharging cycle of the battery as a function of the average SoC and the SoC swing as proposed in [21]. For one charging/discharging cycle, if the average SoC is denoted by SoC_{avg} and the SoC swing is denoted by SoC_{sw}, then the SoH degradation in this cycle, denoted by $D_{SoH,cycle}$ can be calculated as

$$\begin{aligned} D_{SoH,cycle} = & SoH \cdot \exp\left[4K_{SoC}\left(SoC_{avg} - 0.5\right)\right] \\ & \cdot \exp\left[K_T\left(T_B - T_{ref}\right) \cdot \frac{T_B}{T_{ref}}\right] \\ & \cdot \left\{K_{co} \exp\left[\left(SoC_{sw} - 1\right)\frac{T_{ref}}{K_{ex} T_B}\right] + \frac{0.2\tau}{\tau_{life}}\right\} \end{aligned} \tag{6}$$

where K_{SoC}, K_{co}, K_{ex} and K_T are battery specific parameters, T_B is the battery temperature, T_{ref} is the reference temperature, τ is the duration of the charging/discharging cycle, and τ_{life} is the calendar life of the battery.

If a battery has gone through a series of charging/discharging cycles, and the SoH degradation of each cycle is calculated as $D_{SoH,cycle}^{(1)}, \ldots, D_{SoH,cycle}^{(M)}$ using Eqn. (6), then the SoH of the battery can be calculated as

$$SoH = 1 - \sum_{m=1}^{M} D_{SoH,cycle}^{(m)} \tag{7}$$

Therefore, if the purchasing price of a battery is denoted by $Price^{bat}$, the value loss of the battery for a given amount of SoH degradation, denoted by $Cost_{SoH}$, can be calculated as

$$Cost_{SoH} = \frac{D_{SoH}}{1 - SoH_{th}} \cdot Price^{bat} \tag{8}$$

where SoH_{th} is the preset SoH threshold level after reaching which the battery should be replaced.

III. Problem Formulation and Solution Method

For the purpose of maximizing the net profit, a data center operator should consider both to increase the revenue by serving the clients and to reduce the capital and operational cost incurred by the data center. The revenue depends on the total number of clients using the service and the fee charged for each client, which is set by the service level agreement (SLA) and is usually a function of the response time of the service request. On the other hand, the expenditure of the data center mainly consists of the amortized cost of its components and the utility cost which depends on the power consumption. Since most parts of the data center including the servers, the network connections, the cooling system, etc., are considered to have constant life expectancy which is irrelevant to the request dispatch, resource allocation, or the battery management policy, we can omit them in our problem formulation.

A. Calculating the power consumption in the multi-level power hierarchy

Seen from the perspective of the power grid, the total power consumption of the data center is comprised of two major parts: 1) the power supply to keep the servers functioning, which includes the power consumption of the servers as well as other supporting components such as the power infrastructure, the network connection, the cooling system, etc., and 2) the power flow into/out of the centralized and decentralized battery cells which serves the purpose of peak power shaving and operational cost minimization. To account for first component which depends on the utilization status of each server, we use the following model to calculate the total power consumption of the j-th server in time period l, which is denoted by $P_{j,l}^{serv}$

$$\begin{aligned} P_{j,l}^{serv} &= k_{\phi,j} \cdot \phi_{j,l} + k_{\lambda,j} \cdot (\phi_{j,l}\mu_j) \cdot \frac{p_{j,l}\lambda_l}{\phi_{j,l}\mu_j} + k_{C,j} \\ &= k_{\phi,j} \cdot \phi_{j,l} + k_{\lambda,j} \cdot p_{j,l} \cdot \lambda_l + k_{C,j} \end{aligned} \tag{9}$$

where $k_{\phi,j}$, $k_{\lambda,j}$, and $k_{C,j}$ are parameters that depend on the power efficiency of the server and the *power usage effectiveness* (PUE) [22] of the data center. In Eqn. (9), the right hand side consists of three terms. The first term is the power consumption related to activating the requested amount of resource, which is proportional to $\phi_{j,l}$, the second term is the power consumption related to processing the service requests, which is proportional to the request arrival rate, and the last term is the static power consumption of the server as well as the constant power overhead. To account for the second component of the total power consumption, we denote the input power for the battery array labeled by (k, m) in time period l by $P_{k,m,l}^{bat}$, and the equivalent energy increasing rate considering rate capacity effect by $P_{k,m,l}^{bat,eq}$. Using the power consumption of servers as modeled in Eqn. (9), the total input power from the power grid in time period l can be calculated iteratively as a function of $P_{j,l}^{serv}$'s and $P_{k,m,l}^{bat}$'s. If the amount of power required by node (k, m) in time period l is denoted by $P_{k,m,l}^{req}$, then $P_{1,1,l}^{req}$ is the total power that is drawn from the power grid. When $k < K$, i.e. except for the case of a leaf node, the following equation holds

$$P_{k,m,l}^{req} = \sum_{S(k+1,m')=m} P_T\left(P_{k+1,m',l}^{req}; \eta_{T,k}\right) + P_{k,m,l}^{bat} \tag{10}$$

where $P_T(P_0; \eta)$ is the function to calculate the input power required from one level higher in the hierarchy in the case that the actual required power is P_0 and the transmission efficiency between the two adjacent level is η, which is defined as follows

$$P_T(P_0; \eta) = \begin{cases} \frac{1}{\eta} P_0, & P_0 \geqslant 0 \\ \eta \cdot P_0, & P_0 < 0 \end{cases} \tag{11}$$

The reason that $P_T(P_0; \eta)$ has different expressions when $P_0 \geqslant 0$ and $P_0 < 0$ is that the power flows in opposite directions in the two cases. And when $k = K$, we have

$$P_{K,m,l}^{req} = P_{m,l}^{serv} + P_{K,m,l}^{bat} \tag{12}$$

Therefore, given the request flow status, the resource allocation scheme, and the battery management policy, one can start from calculating the values of $P_{K,m,l}^{req}$'s using Eqn. (12). And once the values of $P_{k,m,l}^{req}$'s of the k-th level are all obtained, one can proceed to calculating the values of $P_{k-1,m,l}^{req}$ by applying Eqn. (10) and (11). This process can be repeated until the value of $P_{1,1,l}^{req}$ is obtained.

From Eqn. (3), (9) and (12), we know that $P_{K,m,l}^{req}$ is a convex increasing function of $P_{k,m,l}^{bat,eq}$'s, $p_{j,l}$'s, and $\phi_{j,l}$'s jointly. Since the transmission efficiency can only take values from $(0, 1)$, the function $P_T(P_0; \eta)$ as defined in Eqn. (11) is a convex increasing function of P_0. Furthermore, since the non-negative weighted sum of convex increasing functions and the convex increasing function of a convex increasing function are both convex increasing functions [23], using the relation specified in Eqn. (10), one can prove using mathematical induction that $P_{1,1,l}^{req}$ is also a convex function of $P_{k,m,l}^{bat,eq}$'s, $p_{j,l}$'s, and $\phi_{j,l}$'s jointly.

B. Problem formulation

To account for both the revenue and the expenditure, we define a cost per day function which can be used as the objective function in the optimization problem, denoted by $Cost_{total}$, as follows

$$\begin{aligned} Cost_{total} = &\sum_l \left(Price_l^{elec} \cdot P_{1,1,l}^{req} \cdot T_l\right) + \sum_{k,m} Cost_{SoH,k,m} \\ &+ k_D \sum_l \left(\bar{R}_l \cdot \lambda_l \cdot T_l\right) \end{aligned} \tag{13}$$

where $Price_l^{elec}$ is the electricity price in time period l, $P_{1,1,l}^{req}$ is calculated using Eqn. (9) – (12), $\bar{R}_l$ is the average request response time as defined in Eqn. (2), k_D is a positive factor which can be set by the data center operator based on the SLA to reflect the revenue loss when the response time increases, and $Cost_{SoH,k,m}$ is the value loss of the battery array (k,m) in one day which is a function of the SoC levels of battery array (k,m) in each time period. The first term on the right hand side addresses the total electricity cost per day, the second term addresses the value loss for the battery per day, and the last term addresses the total revenue loss per day.

In order to minimize the cost function as specified in Eqn. (13), we can determine the request flow pattern, the amount of allocated resource of each server, and the charging/discharging policies for the battery arrays based on the incoming workload and the time-of-day utility price. Therefore, the optimization problem can be formulated as follows:

Find $p_{j,l}$'s, $\phi_{j,l}$'s, $P_{k,m,l}^{bat,eq}$'s, $SoC_{k,m,l}$'s

Minimize $Cost_{total}$

Subject to

$$\textstyle\sum_j p_{j,l} = 1 \qquad \forall l \qquad (14)$$

$$p_{j,l} \cdot \lambda_l \leqslant \phi_{j,l} \cdot \mu_j, \qquad \forall j,l \qquad (15)$$

$$SoC_{k,m,l+1} = SoC_{k,m,l} + \frac{P_{k,m,l}^{bat,eq} \cdot T_l}{V_{k,m}^{bat} \cdot C_{k,m}^{full}}, \qquad \forall k,m,l \qquad (16)$$

$$SoC_{k,m,l} \geqslant SoC_{th}, \qquad \forall k,m,l \qquad (17)$$

$$SoC_{k,m,l} \leqslant 1, \qquad \forall k,m,l \qquad (18)$$

$$SoC_{k,m,1} = SoC_{k,m}^{begin}, \qquad \forall k,m \qquad (19)$$

$$SoC_{k,m,L+1} = SoC_{k,m}^{end}, \qquad \forall k,m \qquad (20)$$

$$p_{j,l} \in [0,1] \qquad \forall j,l \qquad (21)$$

$$\phi_{j,l} \in [0,1] \qquad \forall j,l \qquad (22)$$

where $p_{j,l}$ is the probabilities that a request is dispatched to the j-th server in time period l, $\phi_{j,l}$ is the proportion of resources in the j-th server that is used to process incoming requests in time period l, $P_{k,m,l}^{bat,eq}$ is the equivalent input power of battery array (k,m). Constraint (14) ensures that all the requests are dispatched to a specific server to be processed. Constraint (15) ensures that each server allocates enough resource to the incoming requests. In Constraint (16), the SoC of each battery array in the next time period is calculated from the SoC of the current time period and the equivalent input power of the current time period. $V_{k,m}^{bat}$ is the terminal voltage of battery array (k,m) and $C_{k,m}^{full}$ is the capacity of battery array (k,m) when it is fully charged that can be calculated from $C_{k,m}^{nom}$ and the SoH of the battery array. Constraints (17) and (18) set the upper bound of the SoC to 100% and the lower bound of SoC to a preset value SoC_{th} such that some energy is reserved for backup in the case of power outages. Constraints (19) and (20) set the SoC level for each battery array at the beginning of the day as well as the desired SoC for each battery array at the end of the day. Constraints (21) and (22) set the domain for $p_{j,l}$'s and $\phi_{j,l}$'s. Please note that although how to set the values for $SoC_{k,m}^{begin}$'s and $SoC_{k,m}^{end}$'s in Constraints (19) and (20) is also an interesting problem, it is beyond the scope of this paper and we consider these values already given.

C. Solution method

One major challenge to solve the problem formulated in Section III-B comes from the term $Cost_{SoH,k,m}$ in Eqn. (13) because the SoH degradation in one day is difficult to calculate using cycle based SoH calculation in Eqn. (6). Since we make the observation that charging/discharging cycles with a larger SoC swing have more significant impact on the SoH of the battery, the SoC change in each day can be approximated as one large charging/discharging cycle between the highest SoC and the lowest SoC observed during the day. Therefore, the SoH degradation can be calculated directly by applying Eqn. (6). The original problem formulation should be modified by adding the average SoC for each battery array, denoted by $SoC_{k,m}^{avg}$'s, and the SoC swing for each battery array, denoted by $SoC_{k,m}^{sw}$'s, as control variables. To be consistent with the chosen average SoC and the SoC swing, the highest SoC and the lowest SoC that battery array (k,m) can have, denoted by $SoC_{k,m}^{hi}$ and $SoC_{k,m}^{lo}$, respectively, can be calculated as follows

$$SoC_{k,m}^{hi} = SoC_{k,m}^{avg} + \frac{SoC_{k,m}^{sw}}{2} \qquad (23)$$

$$SoC_{k,m}^{lo} = SoC_{k,m}^{avg} - \frac{SoC_{k,m}^{sw}}{2} \qquad (24)$$

And the following constraint should be added to the original problem formulation

$$SoC_{k,m}^{lo} \leqslant SoC_{k,m,l} \leqslant SoC_{k,m}^{hi}, \quad \forall k,m,l \qquad (25)$$

$$SoC_{k,m}^{avg} \in [0,1], \qquad \forall k,m \qquad (26)$$

$$SoC_{k,m}^{sw} \in [0,1], \qquad \forall k,m \qquad (27)$$

where Constraint (25) sets the bound for $SoC_{k,m,l}$ with specific average SoC and SoC swing, and Constraints (26) and (27) set the domain for $SoC_{k,m}^{avg}$'s and $SoC_{k,m}^{sw}$'s.

With the aforementioned approximation, one can make the following observations from the formulated problem. 1) All equality constraints and inequality constraints are affine functions of the control variables. 1) The first two terms in the expression for $Cost_{total}$ as defined in Eqn. (13), i.e. $\sum_l \left(Price_l^{elec} \cdot P_{1,1,l}^{req} \cdot T_l\right)$ and $\sum_{k,m} Cost_{SoH,k,m}$ are convex functions of the control variables, which can be proved using Eqn. (3), (6), (8), (9) – (12). 3) The third term in Eqn. (13), i.e. $k_D \sum_l \left(\bar{R}_l \cdot \lambda_l \cdot T_l\right)$ is a convex function of all control variables except for $p_{j,l}$'s, and at the same time, it is a convex function of all control variables except for $\phi_{j,l}$'s.

Based on these three observations, we propose an efficient approach that solves the problem in two phases, namely the request dispatch phase and the resource allocation phase. In the request dispatch phase, we set the values of $\phi_{j,l}$'s as constant and solve the optimization problem subject to all constraints except for Constraint (22). And in the resource allocation phase, we set the values of $p_{j,l}$'s as constant and solve the optimization problem subject to all constraints except for Constraints (14) and (21). Since the problems in both phases are convex, standard convex optimization tools, e.g. CVX [24], can be used to solve the problems. To achieve better results, the two optimization phases can be performed iteratively until the solution converges.

IV. Experimental Results

In this section, we present the simulation results of the proposed optimization framework on selected data center configurations. Please note that for clarity of illustration, we use normalized values rather than absolute values of parameters related to time, power consumption, and/or price.

We use a 3-level power hierarchy which corresponds to the data center level, the rack level and the server level. The data center level node is connected to 2 nodes at the rack level, and each node at the rack level is connected to 4 nodes at the server level, which then connects to a group of servers. The capacity of battery arrays at the data center level, the rack level, and the server level is set to 4, 2, and 1, respectively. The transmission efficiency between two adjacent levels is set to 0.9. According to [25], we set the parameters in the SoH degradation model in Eqn. (6) as follows

$$\begin{aligned} K_{SoC} = 0.916 \quad K_{co} = 3.66 \times 10^{-5} \\ K_{ex} = 0.717 \quad K_T = 0.0693 \end{aligned} \tag{28}$$

The initial SoH is set to 100% and the temperature is set to 300K. The Peukert factors, γ_C and γ_D, as in Eqn. (3) are both set to 1.1, and the charging and discharging efficiency of the batteries, η_C and η_D are both set to 0.85. The terminal voltage of each battery, V_{bat}, is set to 1, and the reference charging/discharging current, I_{ref}, is set to $\frac{1}{20}$ of the battery capacity. SoH_{th} as in Eqn. (8) is set to 80% for each battery array, and the amount of reserved SoC as in Eqn. (17) is set to 5%. The initial SoC and the required SoC at the end of the day are set to 50% for each battery array. The price per unit amount of battery capacity is set to 200. Each day is divided into 24 time periods with equal length, each having a normalized duration of 1. The request arrival rates, λ_l's, are randomly selected from $[\lambda_0, 2\lambda_0]$ (which matches the workload fluctuation in Google cluster dataset[2]) where λ_0 is a reference request arrival rate that can be changed in different rounds of simulations. The electricity prices, $Price_l^{elec}$'s, are randomly selected from $[0.05, 0.15]$. To account for the heterogeneity of the servers, we also use random values for parameters in the delay model (Eqn. (1)) and the power consumption model (Eqn. (9)). The maximum processing rate of each server, μ_j, follows a uniform distribution between $[1.0, 2.0]$. The power consumption factors, $k_{\lambda,j}$, $k_{\phi,j}$, and $k_{C,j}$, follow uniform distributions between $[1.0, 2.0]$, $[1.0, 2.0]$, and $[0.2, 0.4]$, respectively.

First, we show how the amount of workload will affect the total cost of the data center. The factor k_D in Eqn. (13) is set to 0.5, and the value of λ_0 varies from 1.0 to 2.0. The simulation results of three different random schemes are shown in Fig. 2. As can be seen from the figure, the total cost per day function is generally an increasing function of the average request arrival rate, which matches the intuition because with more incoming requests, the servers will consume more power to finish processing each request in time, and the revenue loss will increase due to a larger number of requests and possibly longer processing delay. When λ_0 is doubled from 1.0 to 2.0, the total cost per day increases by 75%.

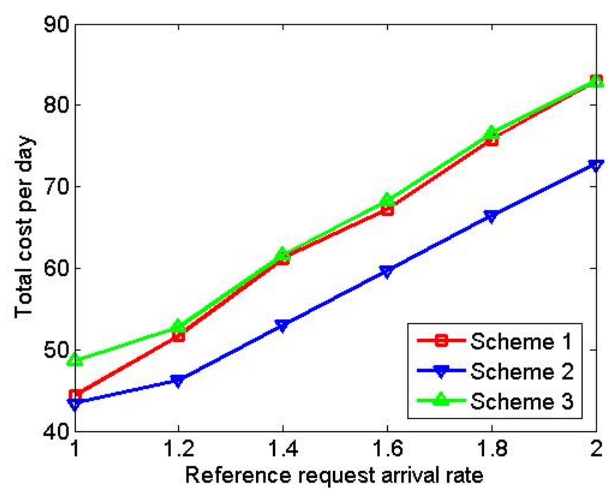

Figure 2. The total cost per day of a data center under different request arrival rate

Then, we show how different components of the total cost as in Eqn. (13) are balanced with different values of k_D. The change of total cost per day as well as the cost resulting from revenue loss with $\lambda_0 = 1.5$ and k_D ranging from 0.25 to 0.75 is shown in Fig. 3. As can be seen from the figure, the contribution of revenue loss increases with higher k_D values. However, the increase in revenue loss is not proportional to the increase in k_D. When k_D increases by 50% from 0.50 to 0.75, the revenue loss only increases by 40.7%, which means that the average delay is reduced at the cost of higher power consumption so that the total cost is minimized. The same is true when k_D decreases by 50% from 0.50 to 0.25, and

[2]https://code.google.com/p/googleclusterdata/

the revenue loss only decreases by 43.7%.

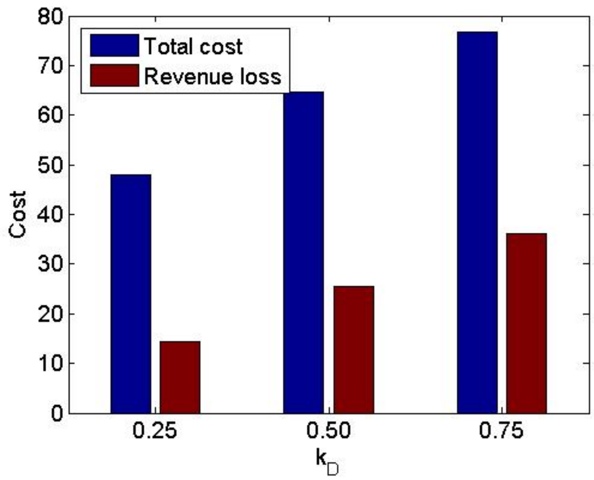

Figure 3. The contribution of revenue loss to the total cost per day

V. Conclusion

In this paper, we address the joint optimization problem of request dispatch, resource allocation, and battery management in a data center with multi-level power hierarchy and energy storage devices. In addition to the efficiency of energy transmission and conversion, the rate capacity effect and the SoH degradation of the battery are also considered, which adds to the accuracy of the power consumption model. Future work includes the incorporation of the battery deployment problem and the study of more efficient power hierarchy.

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