Maximum Power Transfer Tracking for a Photovoltaic-Supercapacitor Energy System *

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ABSTRACT

It is important to maintain high efficiency when charging electrical energy storage elements so as to achieve holistic optimization from an energy generation source (e.g., a solar cell array) to an energy storage element (e.g., a supercapacitor bank). Previous maximum power point tracking (MPPT) methods do not consider the fact that efficiency of the charger varies depending on the power output level of the energy generation source and the state of charge of the storage element. This paper is the first paper to optimize the efficiency of a supercapacitor charging process by utilizing the MPPT technique and simultaneously considering the variable charger efficiency. More precisely, previous MPPT methods only maximize the power output of the energy generation source, but they do not guarantee the maximum energy is stored in the energy storage element. Note that the load device takes its energy from the storage element so it is important to maximize energy transfer from the source into the storage element. We present a rigorous framework to determine the optimal capacitance of a supercapacitor and optimal configuration of a solar cell array so as to maximize the efficiency of energy transfer from the solar cells into a bank of supercapacitors. Experimental results show the efficacy of the proposed technique and design optimization framework.

Categories and Subject Descriptors

J.6 [**Computer-aided engineering**]: Computer-aided design (CAD)

General Terms

Algorithms, Design

Keywords

Maximum power transfer tracking, Photovoltaic, Supercapacitor

1. INTRODUCTION

Maximum energy efficiency can be achieved by a global optimization from the energy generation source to a load device which consumes the energy. Energy optimization of power consuming electronic devices including microprocessors, memory, storage, communication and networking chips has been an active area of research. More recently, energy efficiency of switching converters and voltage regulators has received some attention. Examples include battery-aware power management [1] and switching converter efficiency-aware power management [2]. By contrast, energy generation and storage elements have not received much attention.

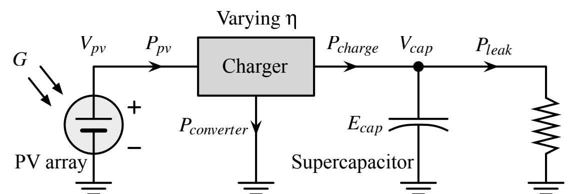

Figure 1: Photovoltaic-supercapacitor energy system.

The power efficiency becomes crucial when the power comes from a renewable source such a solar cell (photovoltaic, or PV for short) or a windmill. The output voltage and power of these power sources are highly variable depending on the current draw. Moreover, the optimal current that maximizes the power also changes with the environmental conditions (solar irradiation or wind intensity). In general, the output impedance of the renewable energy sources changes based on the surrounding environment. The maximum power point tracking (MPPT) methods dynamically adjust the output current to match the output impedance so that the maximum amount of power can be drawn from the power generating device [3, 4, 5]. Non-ideal characteristics of supercapacitors must be considered when designing solar-harvesting circuits as shown in [6].

Practical deployment of a renewable energy source mandates an electrical energy storage element to compensate the output power fluctuation of the renewable source. Generally, the energy storage element of a renewable energy source experiences very frequent charge and discharge phase changes. Supercapacitors, also known as electrolytic double layer capacitors, are one of the most promising energy storage elements for this application because of i) orders-of-magnitude longer cycle life compared with ordinary batteries, ii) high power rating, iii) no limitation in deep cycle use, and iv) very low negative environmental impact. Supercapacitors and PV modules are an excellent combination because the energy storage element typically performs deep-cycle discharge every night.

This paper is the first to introduce an MPPT-like technique for renewable energy sources considering the charger efficiency. Although we demonstrate our proposed technique on a combination of PV modules and supercapacitors, the proposed energy optimization methodology is not limited to them.

Figure 1 illustrates a simplified schematic diagram from energy generation to storage. The total system efficiency enhancement seeks to maximize the power that is transferred into the supercapacitor, P_{charge}. In contrast, previous MPPT work aim to maximize the power output from the PV module, P_{pv}. If the efficiency of the charger η is constant, maximizing P_{pv} also maximizes P_{charge}. However, as shown in Figure 2, typical switching charger efficiency for superca-

*This work is supported by the Brain Korea 21 Project, IC Design Education Center (IDEC), and Mid-career Researcher Program through NRF grant funded by the MEST (No. 2010-0017680). The ICT at Seoul National University provides research facilities for this study.

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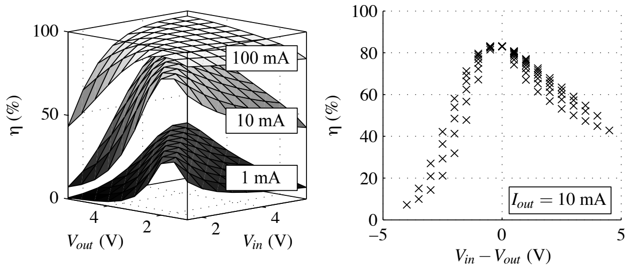
ISLPED'10, August 18–20, 2010, Austin, Texas, USA.



(a) Input voltage, output voltage and output current (b) Input and output voltage difference

Figure 2: Factors that affects the switching converter efficiency.

pacitors changes dramatically as a function of its input and output voltage difference, i.e., the PV module voltage and the supercapacitor terminal voltage.

We formulate the charging efficiency η based on a tight interaction between the supercapacitor and the charger. A supercapacitor is a type of capacitor, i.e., its terminal voltage is a linear function of its state of charge in such a way that $Q = C \cdot V_{cap}$ where Q, C and V_{cap} denote the state of charge, capacitance, and terminal voltage of the supercapacitor, respectively. The charger efficiency varies from 10% to 80% as a function of its input-output voltage difference and, in turn, the supercapacitor's state of charge. Because the charger efficiency η is not constant, conventional MPPT methods do not any longer guarantee the maximum P_{charge}. In other words, even if conventional MPPT achieves the maximum power drawn from the PV modules, a large portion of power is simply dissipated as heat in the charger, and never goes to the supercapacitor. Furthermore, determination of C is constrained by both the amount of required energy storage and the charger efficiency. When we consider variable η, the MPPT technique does no longer result in a closed-form solution. Indeed no systematic optimization framework for MPPT with consideration of a variable η exists. This paper presents a systematic design framework to maximize the cost-efficiency for a renewable energy generation and its storage element considering variable charger efficiency η, which in turn significantly impacts the overall PV-supercapacitor system efficiency.

The remainder of the paper is organized as follows. First we explain how the overall system efficiency is affected by the non-ideal characteristics of the PV module, efficiency variation of the switching-mode supercapacitor charger, and characteristics of supercapacitors. Then we present a new charging method to maximize the actual power that goes from the PV module into the supercapacitor, P_{charge}. Next we explore how the delivered power changes by the design-time decision parameters. Finally, we introduce a cost-effective and energy-efficient design framework for a renewable energy generation and its storage element. Using the proposed framework, we demonstrate the enhancement of the maximum energy generated by a PV module and stored in a supercapacitor in a day, in comparison with conventional methods.

2. POWER MODELS

2.1 Photovoltaic Module

A PV cell is typically modeled as a circuit shown in Figure 3. The general I-V characteristic of a PV module may be written as

$$I_{pv} = I_L - I_0 \left(e^{q(V_{pv}+I_{pv}\cdot R_s)/n_s AkT} - 1 \right) - \frac{V_{pv} + I_{pv} \cdot R_s}{R_{sh}}, \quad (1)$$

where n_s is the number of connected PV cells in series, I_L and I_0 denote PV cell current and diode saturation current, R_s and R_{sh} are panel series resistance and panel parallel (shunt) resistance, respectively, and A is the diode quality factor.

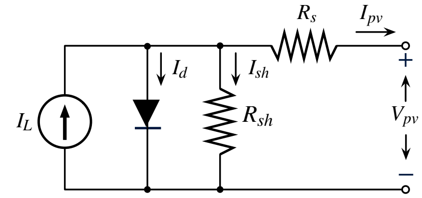


Figure 3: PV module model.

A previous work [7] extracts the panel parameters from datasheet values, comprising the short-circuit current I_{sc}, open-circuit voltage V_{oc}, current and voltage levels at the maximum power point, I_{mpp} and V_{mpp}, and temperature coefficients. An iterative method can determine these parameters, but takes a long time to converge. Based on this model, we employ the Newton-Raphson method for better convergence, considering all the parameters.

2.2 Switching Converter

Due to the wide range of the terminal voltage variation of the PV module and the supercapacitor, direct connection of the supercapacitor to the PV module may prevent the PV module from operating at its energy-efficient point [4, 5]. Generally, a buck-boost switching converter, which can generate a desired output voltage regardless of the input voltage, is used to extract more power even with weak irradiance (low light intensity) in the evening.

The charger efficiency is given by

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{charge}}{P_{pv}} = \frac{P_{pv} - P_{converter}}{P_{pv}}, \quad (2)$$

where $P_{converter}$ is the power consumed by the converter, which comprises the conduction loss, gate-drive loss, and controller power dissipation. If $P_{converter}$ is constant, maximizing the extracted power P_{pv}, which is what the MPPT techniques do, guarantees maximizing the transferred power P_{charge}. However, the power dissipation of a switching converter is strongly dependent on the input voltage V_{in} , output voltage V_{out} , and output current I_{out}. We borrow the energy model of the switching converter from [2]. However, our method is not restricted to a specific analytical model or a product of a switching converter because the optimization method does not exploit specific characteristics of the analytical model.

Figure 2 shows the varying efficiency η for the charger model. Typically, the efficiency is satisfactory when the input and output voltages are similar in magnitude. Considering that the efficiency ranges from 10% to 80%, it is absolutely unwarranted to assume constant energy conversion efficiency in a solar-powered system with a supercapacitor storage element. If we design a MPPT method assuming a constant η, a large portion of the power is simply dissipated as heat in the charger, even though we draw the maximum power from the PV module.

2.3 Supercapacitor

Supercapacitors have superior characteristics over batteries in terms of cycle efficiency and cycle life [5]. Their cycle efficiency, which is defined as the ratio of the energy input to the energy output of an energy storage element, reaches almost 100%.

The terminal voltage of a supercapacitor is a linear function of its state of charge, which is given by

$$V_{cap}(t) = \frac{Q(t)}{C} = \sqrt{\frac{2 \cdot E_{cap}(t)}{C}}, \quad (3)$$

where E_{cap} is the energy stored in the supercapacitor, and C is the capacitance. E_{cap} increases or decreases dynamically as charge or discharge operations are performed. The same is true for V_{cap}, whose variation is much higher than the terminal voltage of an ordinary bat-

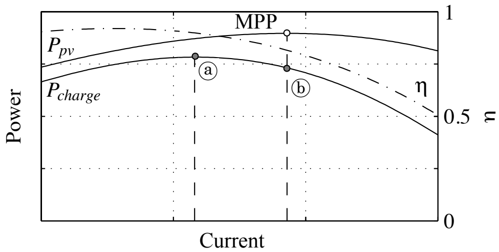


Figure 4: Conceptual example of current draws that maximize P_{charge} and P_{pv} when efficiency varies. $P_{charge} = \eta \cdot P_{pv}$.

tery, and therefore, the charger efficiency of a supercapacitor varies more significantly compared with that of a battery.

A primary disadvantage of a supercapacitor is its larger self-discharge rate compared with that of an ordinary battery. A supercapacitor may lose more than 10% of its stored energy per day even if no load is connected to it. Voltage decay after Δt time is given by

$$V_{cap}(t+\Delta t) = V_{cap}(t) \cdot e^{-\Delta t/\tau}, \tag{4}$$

where τ is the self-discharge time constant. A supercapacitor cannot be used as long-time electrical energy storage, e.g., a few days or more. However short storage times up to a day or so is possible.

2.4 Integrated System Energy Model

This subsection derives the overall system energy model aggregating each component's efficiency. Figure 1 shows an equivalent circuit of the system. We can rewrite terms in (2) as $P_{in} = P_{pv} = V_{pv} \cdot I_{pv}$ and $P_{out} = P_{charge} = V_{cap} \cdot I_{charge}$. The amount of energy stored in the supercapacitor E_{cap} can be calculated for a time slot Δt using a discrete-time energy model:

$$E_{cap}(t+\Delta t) = E_{cap}(t) + \Delta t \cdot \left(P_{charge}(t) - P_{leak}(t)\right), \tag{5}$$

where P_{charge} is the power transferred to the supercapacitor through the charger, and P_{leak} is the leakage power from the supercapacitor due to self-discharge. We assume that the solar irradiance profile G is given as a function of time t from sunrise $t_{sunrise}$ to sunset t_{sunset}. All these models are implemented and simulated in Matlab. We assume G is a sine function with the peak achieved at noon time. Note that this sinusoidal solar irradiance model result in only 5.2 W/m^2 of RMS error when compared with the irradiance profile measured on a sunny day with around 800 W/m^2 of maximum irradiance in Phoenix, AZ on March 17, 2010 [8].

3. ENERGY-EFFICIENT CHARGING

3.1 Conventional MPPT Method

The MPPT technique first identifies the maximum power point (MPP) to draw the maximum P_{pv} and continuously keeps track of this point against the irradiance variation and/or load impedance variation. There are many previous contributions that achieve MPPT. Perturb & observe (P&O) method and incremental conduction method identify the MPP by generating a slight change in I_{pv} and observing the change in P_{pv} [9]. Ripple correlation control method finds the MPP using the time derivative of I_{pv} and P_{pv} [3]. As for economical implementations of MPPT, a small pilot cell or a linear relationship of the MPP to the open-circuit voltage or short-circuit current can help estimate the MPP [4, 5].

As mentioned in Section 1, to maximize the amount of energy stored in the supercapacitor, we should maximize $P_{charge} = \eta \cdot P_{pv}$. Indeed conventional MPPT methods maximize P_{pv} expecting that P_{charge} is also maximized. However, MPPT without considering the charger efficiency may result in low transferred power, as shown in Figure 4. More precisely, conventional MPPT simply finds the operating point that results in maximum P_{pv} regardless of the charger efficiency at that point. In the figure, this point is ⓑ. In reality we are interested in maximizing P_{charge}, which is achieved at point ⓐ in the figure. This is because at point ⓐ, although the current I_{pv} is offset from the current of the MPP, since η is high enough, P_{charge} at this point is larger than P_{charge} at the MPP, i.e., point ⓑ. Consequently, MPPT and maximum efficiency tracking of the charger should be considered at the same time. Because η is affected by both the PV module voltage and the supercapacitor voltage, we have to take the status of the supercapacitor into account when finding the optimal operating point that maximizes P_{charge}.

3.2 Problem Definition

We harvest the solar energy during daytime and typically store some of the energy for use during the nighttime. To focus on energy storage efficiency and avoid divergence from the main context and purpose of the present paper, we assume that there is no energy usage during the daytime. The amount of energy to be stored at the end of the day, E_{req} is given as a design requirement. The objective of the proposed design optimization is to derive a cost-effective and energy-efficient design of a solar energy generation and storage system to achieve this requirement.

Cost-effectiveness is defined as the minimum number of PV modules that meet the energy storage requirement, E_{req}. Energy efficiency means maximizing the amount of energy that is eventually stored in the supercapacitor at the end of the day with the same number of PV modules. These two optimizations are coupled together; if we enhance the efficiency, we may use small number of PV modules while satisfying E_{req}.

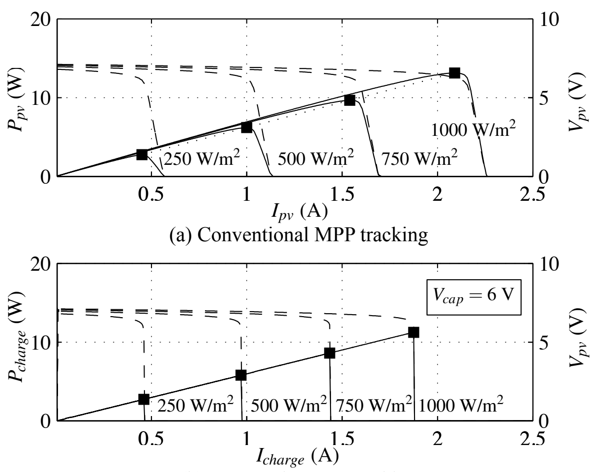
The amount of energy stored during the daytime is given by

$$E_{cap}(t_{sunset}) = E_{cap}(t_{sunrise}) + \int_{t_{sunrise}}^{t_{sunset}} \left(P_{charge}(t) - P_{leak}(t)\right) dt. \tag{6}$$

We must consider another constraint, i.e., feasibility. Commercially available switching converters and chargers have the maximum voltage rating around 30 VDC, and we must thus limit the maximum voltage of the supercapacitor bank and the PV modules in series in Figure 1. We consider identical PV arrays with the configuration of n in series and m in parallel. Due to this regularity, we can operate all the PV cells at the identical operating condition and maintain uniform energy efficiency for all cells.

The maximum amount of energy that can be stored in a supercapacitor bank is proportional to $C \cdot {V_{cap\_max}}^2$, where V_{cap_max} is the maximum voltage rating of the supercapacitor bank. We can change C and V_{cap_max} by connecting more unit-sized supercapacitors in parallel and series, respectively. However, the total number of unit-sized supercapacitors necessary to store a certain amount of energy does not change no matter how we configure the series and parallel connections. Therefore the supercapacitor cost, which is proportional to the total number of unit-sized supercapacitors, is not the optimization objective that we consider. We first derive the amount C that results in maximum energy harvesting for the given E_{req}, and next, we determine V_{cap_max} using (3).

To make a long story short, smaller voltage difference between the PV array and supercapacitor bank in Figure 1 achieves better charger efficiency. Thus the key optimization method is to control the supercapacitor voltage by adjusting C, because the PV array voltage is determined by solar irradiance, which is not controllable. For example, if the supercapacitor bank capacitance is too small, the supercapacitor bank voltage rises quickly and goes way higher than the nominal PV array voltage. On the other hand if the supercapacitor bank capacitance is too big, the supercapacitor bank voltage does not rise much and remains far lower than the nominal PV array voltage.



(a) Conventional MPP tracking

(b) Proposed MPTP tracking

Figure 5: V-I curves of a 7×10 PV array.

Determination of the supercapacitor bank capacitance is also coupled with the PV array configuration. We must jointly optimize how many series and parallel connections of PV cells within a PV array should be established given the supercapacitor capacitance.

We name the overall PV-supercapacitor system efficiency optimization as maximum power transfer tracking (MPTT) design. The MPTT design can be formally described as follows:

- Given: The energy requirement E_{req} and the maximum voltage rating V_{rating}.
- Prerequisite: PV module characteristics, charger efficiency model, and solar irradiance profile G.
- Objective: i) Find a $n \times m$ PV array configuration which minimizes $n \cdot m$ while meeting $E(t_{sunset}) \geq E_{req}$; and ii) given the $n \times m$ configuration, find a supercapacitor bank capacitance C that maximizes the energy efficiency while meeting $V_{cap}(t_{sunset}) \leq V_{rating}$.

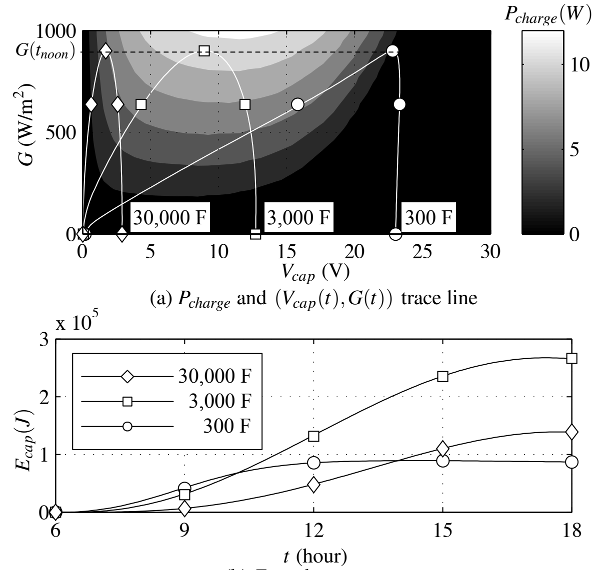
It is possible to have $V_{cap}(t)$, for some t, higher than $V_{cap}(t_{sunset})$ due to the self-discharge. However, we assume this is negligible, as we will see in the following section for a half-day storage.

3.3 Maximum Power Transfer Point

Figure 5(a) shows V_{pv} and P_{pv} variations according to I_{pv} with four different irradiance values. Even with the same irradiance, we can see a significant change in P_{pv}. The MPPT methods discussed in Section 3.1 can be used to find the maximum P_{pv} regardless of the irradiance.

However, if we take the charger efficiency into account, the *P-I* curve we have to consider is the P_{charge}-I_{charge} curve shown in Figure 5(b), rather than the P_{pv}-I_{pv} curve of Figure 5(a). Since the x-axis is I_{charge}, the y-axis P_{charge} is linearly proportional to the I_{charge} when V_{cap} is given. The maximum P_{charge} values, marked by squares, are the maximum power transfer (MPT) points. Beyond this point, further increment of I_{charge} causes a rapid drop of V_{pv}. The V_{pv} values at the MPT points are not the same as the V_{pv} values at the MPP, and P_{charge} at the MPT point is slightly lower than P_{pv} at the MPP because $\eta < 1$.

Our proposed MPTT keeps tracking (V_{pv}, I_{pv}) which may be slightly different from that of conventional MPPT, to guarantee the maximum amount of power transferred to the load at all times rather than the maximum power extracted from the PV module. Note that the proposed MPTT always outperforms the conventional MPPT in terms of net energy delivery to the load regardless of environmental conditions.



(a) P_{charge} and $(V_{cap}(t), G(t))$ trace line

(b) E_{cap} change over t

Figure 6: Energy accumulation trace of a 7×10 PV array.

P_{charge} at the MPT is determined not only by G, but also by the current value of V_{cap}. Figure 6(a) is the surface that consists of the maximum P_{charge} values of a 7×10 PV array in the $G - V_{cap}$ domain. We may draw a trace of $\left(V_{cap}(t), G(t)\right)$ pairs on this surface with the solar irradiance set to a meaningful value in $t \in [t_{sunrise}, t_{sunset}]$. For instance, the white lines are the traces when C is 300 F, 3,000 F or 30,000 F. For illustration purpose, we set $G(t_{noon}) = 900$ W/m^2. Initially, $V_{cap}(t_{sunrise}) = 0$ V. From (5), the value of $P_{charge} - P_{leak}$ is the gradient of E_{cap}. Figure 6(b) shows the supercapacitor's energy, E_{cap} as t elapses. Right after the sun rises, G is low and P_{charge} is also low, and E_{cap} increases slowly. Generally, with a reasonable supercapacitor of C, P_{charge} has the maximum values during the day, and E_{charge} increases most rapidly at noon ($t = t_{noon}$). The sampling points in Figures 6(a) and (b) are matched with each other in terms of t. E_{cap} slightly decreases in the evening due to leakage of the supercapacitor such that $P_{leak} > P_{charge}$ in the evening.

4. DESIGN OPTIMIZATION

4.1 PV Array and the P_{charge} Surface

The configuration of the PV array determines the shape and magnitude of the P_{charge} surface in the $G - V_{cap}$ domain. The total number of PV modules is $N = n \cdot m$, which determines the magnitude of the P_{charge} surface. Evidently the more PV modules are used, the higher power can be achieved. We define P_{peak} and V_{opt} of a P_{charge} surface as

$$P_{peak} = \max_{\forall V_{cap}} \left(P_{charge}\left(G(t_{noon}), V_{cap}\right)\right), \tag{7}$$

$$V_{opt} = \arg\max_{V_{cap}} \left(P_{charge}\left(G(t_{noon}), V_{cap}\right)\right), \tag{8}$$

which gives the maximum possible power that goes to the supercapacitor and its corresponding condition, with an $n \times m$ PV array. The values of n and m determine the location of P_{peak} and V_{opt}. As n increases, V_{opt} increases almost linearly. To achieve the maximum P_{charge} when G is the maximum ($t = t_{noon}$), $V_{cap}(t_{noon})$ should be equal to V_{opt} such that $P_{charge}(t_{noon}) = P_{peak}$.

4.2 Supercapacitor Size and Harvested Energy

It is shown in Figure 7 that V_{cap} is a critical factor for P_{charge}. From (3), V_{cap} is inversely proportional to C. Therefore the deter-

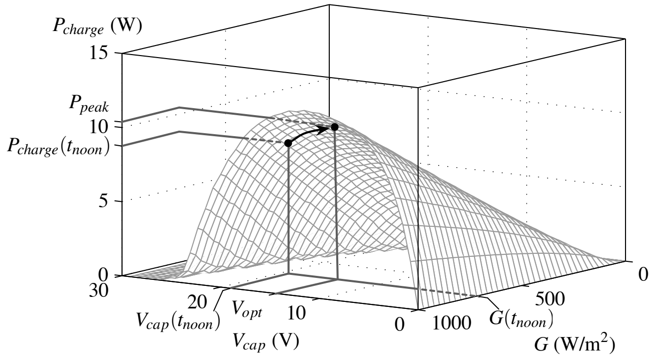


Figure 7: Maximum P_{charge} surface of a 7×10 PV array in GV_{cap} domain.

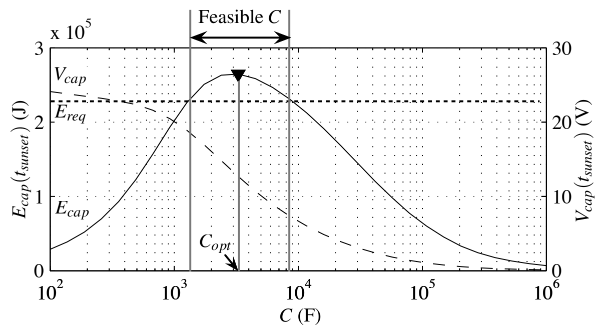


Figure 8: Accumulated energy $E_{cap}(t_{sunset})$ and corresponding $V_{cap}(t_{sunset})$ of a 7×10 PV array.

mination of C is very important for maximizing P_{charge}, and in turn, the accumulated energy, E_{cap}. Figure 8 shows the total amount of accumulated energy, $E_{cap}(t_{sunset})$, as a function of C. It turns out that neither a small nor a large C is energy efficient. This implies that an ad-hoc decision on C may result in a poor energy harvesting. Each curve in Figure 6(a) gives us more intuition about this result. A 3,000 F C is close to the energy optimal. If C is too small (300 F), V_{cap} increases too rapidly and V_{cap} is very different from V_{opt} when G is the maximum. Thus, the curve cannot arrive in the high P_{charge} region. On the other hand, if C is too large (30,000 F), V_{cap} increases too slowly and V_{cap} is again very different from V_{opt} when G is the maximum. Thus again the curve cannot arrive in the high P_{charge} region.

Therefore it is important to determine C so that the system remains in the high-P_{charge} region for a longer period of time. For a symmetrical irradiation profile in a day, we make $P_{charge}(t_{noon}) = P_{peak}$ by adjusting C to guarantee $V_{cap}(t_{noon}) = V_{opt}$. Based on the fact that $V_{cap}(t_{noon}) = V_{opt}$ and using the estimated $E_{cap}(t_{noon})$, the energy-optimal C is calculated by (3).

4.3 Systematic Design Optimization

A naive brute force method to find the optimal design is that first we obtain P_{charge} surfaces for all $n \times m$ PV array configurations, and then evaluate E_{cap} for all C. However, obtaining P_{charge} surfaces as in Figure 7 is very time consuming because of the numerical iterations needed to reach the convergence point of the PV model and charger models. Furthermore, as can be seen in Figure 6(a), each curve passes only a part of the surface, which makes it pointless to calculate P_{charge} for all (V_{cap}, G) pairs.

Based on the observation in Section 2.2 whereby a switching converter exhibits a higher efficiency when the input and output voltages are similar to each other, we develop Algorithm 1 that efficiently derives the near-optimal values of n, m, and C when E_{req}, V_{rating}, and G are given. The objective of this algorithm is to derive the minimum $n \times m$ and optimal C. Since the supercapacitor cost is determined by E_{req}, C is not to be minimized, but to be optimized for harvesting the largest amount of energy. This algorithm requires that the PV model and the charger efficiency model be characterized *a priori*.

Algorithm 1: Design optimization

Input: (energy requirement E_{req}, voltage rating V_{rating}, irradiance profile G, theoretical maximum energy E_{mpp}, noon time t_{noon})
Output: (optimal PV module size n_{opt} and m_{opt}, optimal capacitance C_{opt}), or `null` if given invalid parameters

1 $N \leftarrow \lceil E_{req} / E_{mpp}(G(t_{noon})) \rceil$
2 $n_{max} \leftarrow \infty$; $m_{max} \leftarrow \infty$
3 **repeat**
4 $\quad V_{opt}(1,N) \leftarrow$ `find_V_opt`$(1,N)$
5 $\quad V_{opt}(N,1) \leftarrow$ `find_V_opt`$(N,1)$
6 $\quad N_{next} \leftarrow \infty$
7 $\quad$ **foreach** $(n,m) \in S = \{(n,m) | (n < n_{max} \vee m < m_{max}) \wedge n \cdot m = N \wedge n,m \in \mathbb{N}\}$ **do**
8 $\quad\quad V_{opt}(n,m) \leftarrow$ `linear_approx`(V_{opt}, n, m)
9 $\quad\quad P_{peak}(n,m) \leftarrow$ `maximum_P`$(V_{opt}(n,m), G)$
10 $\quad\quad E_{cap}(n,m) \leftarrow$ `estimate_E_cap`$(P_{peak}(n,m), G)$
11 $\quad\quad C(n,m) \leftarrow 2 \cdot \left(E_{cap}(n,m)/2\right) / V_{opt}(n,m)^2$
12 $\quad\quad V_{cap}(n,m) \leftarrow \sqrt{2 \cdot E_{cap}(n,m)/C(n,m)}$
13 $\quad\quad$ **if** $V_{cap}(n,m) > V_{rating}$ **then**
14 $\quad\quad\quad n_{max} \leftarrow \max(n_{max}, n)$; $m_{max} \leftarrow \max(m_{max}, m)$
15 $\quad\quad\quad$ **if** $m = N$ **then**
16 $\quad\quad\quad\quad$ **return** `null`
17 $\quad\quad$ **else if** $E_{cap}(n,m) < E_{req}$ **then**
18 $\quad\quad\quad N_{next} \leftarrow \min\left(N_{next}, N + \lceil N \cdot (1 - E_{cap}(n,m)/E_{req}) \rceil\right)$
19 $\quad N \leftarrow N_{next}$
20 **until** $\exists (n,m)$ *such that* $E_{cap}(n,m) \geq E_{req}$ *and* $V_{cap}(n,m) \leq V_{rating}$
21 $(n_{opt}, m_{opt}) \leftarrow$ `maximum_E_cap`(E_{cap})
22 $C_{opt} = C(n_{opt}, m_{opt})$
23 **return** $(n_{opt}, m_{opt}, C_{opt})$

We first calculate the minimum feasible number of PV modules by dividing E_{req} by E_{mpp}, which is the theoretical maximum energy that can be extracted by MPP with the maximum G (Line 1). And temporary variables are initialized subsequently. We find V_{opt} values for the two extreme cases such that all PV modules are connected in parallel or in series (Lines 4 and 5).

Based on the observation in Section 4.1, we may linearly approximate V_{opt} for other $n \times m$ configurations from the two extreme cases (Line 8). We estimate the total harvested energy by (5) and $P_{charge}(t) = P_{peak} \cdot G(t)/G(t_{noon})$ where P_{peak} corresponds to the approximated V_{opt}, considering leakage (Line 10). The corresponding C and the maximum V_{cap} are derived by equations on Lines 11 and 12. In each iteration, we prune a large portion of possible configurations that do not satisfy the voltage rating constraint from the current N and larger N (Lines 13). Even the all-parallel configuration may have V_{cap} that exceeds V_{rating} if the given V_{rating} is too low. We stop iteration in such a case (Line 15). If E_{cap} is less than E_{req}, we increase N by the ratio of insufficient energy (Line 18). This is repeated until we find a feasible configuration (Line 20). We choose the configuration that has the maximum E_{cap} among all feasible configurations (Line 21). This algorithm is scalable enough to accommodate large-scale applications due to judicious calculation of N as well as effective pruning.

Table 1: Energy efficiency of the conventional MPPT and suggested MPTT methods.

$n \times m$	C (F)	Tracking method	V_{cap} (V)	E_{cap} (J)	Normalized E_{cap} (%)
5×5	2,378	MPTT	9.0	96,342	100.0
	2,378	MPPT	8.9	93,451	97.0
	23,780		2.2	59,170	61.4
	238		11.0	14,404	15.0
12×2	874	MPTT	15.2	101,545	100.0
	874	MPPT	14.8	95,795	94.3
	8,740		4.2	77,261	76.1
	87		19.1	15,823	15.6

5. EXPERIMENTAL RESULTS

We use Linear Technology LTC3531 buck-boost converter as the charger model, and the Spectrolab GaAs/Ge single junction PV module of $A = 10$ cm^2, which has $V_{oc} = 1.025$ V, $I_{sc} = 0.305$ A, and $P_{mpp} = 0.257$ W at $G = 1353$ W/m^2 in the experiment. We assume that, without loss of generality, the sun rises at 6:00 and sets at 18:00, and $G(t_{noon}) = 900$ W/m^2. We assume 10% self-discharge rate per day for the supercapacitor, which is a typical value for commercially available supercapacitors.

First we show the energy efficiency of the proposed MPTT method compared with conventional MPPT method. Table 1 shows the accumulated energy E_{cap} at the end of the day for various PV array and supercapacitor configurations, which are operated by the MPPT and MPTT methods. The capacitance C of the MPTT case is the theoretical optimum for each given $n \times m$. Most importantly, conventional MPPT methods have no concept of the efficiency-optimal C and PV array configuration, and any C value and any PV array configuration are supposed to yield the same amount of harvested energy. Thus, it is not surprising to have a C value and a PV array configuration for conventional MPPT that yield very poor charging efficiency. We compare the proposed MPTT with conventional MPPT for different C values and PV array configurations. When using the optimal capacitance, MPTT shows more than 6x harvested energy over a poorly configured conventional MPPT as Table 1 shows. The results for poorly configured MPPTs are not embellished because conventional MPPT does not care about the charger loss caused by improper C value. More interesting result is that even the accidentally optimal configuration of conventional MPPT is up to 5.7% less efficient than the proposed MPTT. This is because MPTT finds the true optimal tracking point considering the charger loss while conventional MPPT draws more power from the PV array but loses even more power in the charger. The energy-efficient configuration of MPPT is coincidence and hard to achieve because conventional MPPT gives no clue for the optimal configuration. Note that the 5×5 configuration harvests less energy than the 12×2 configuration while using one more PV cell even the MPTT is applied, and therefore it is not an optimal design. Algorithm 1 can be used to effectively find the optimal design avoiding such a case.

We show the accuracy of Algorithm 1 in terms of actual cost and energy. Recall that Algorithm 1 tries to find the near-optimal value by a heuristic approach to make the computational complexity reasonable. Table 2 is the comparison between designs derived by the suggested algorithm and the optimal design found by exhaustive search for various E_{req} and V_{rating} values. For all cases, we notice that the negative error is less than 2%, which is quite reasonable in light of the typical device tolerance used in commercial circuits. This error is mainly due to the fact that the estimation of E_{cap} is based on the observation that the trace on the P_{charge} curve may be approximated by a sinusoidal waveform. With this approximation, we cannot guarantee a positive or a negative bound on the error. However, we expect to be able to reduce the error by employing other approximations such as a Gaussian fit.

Table 2: Energy harvesting result of designs by Algorithm 1 and exhaustive search (ES).

E_{req} (J)	V_{rating} (V)	Opt. method	$n \times m$	C_{opt} (F)	E_{cap} (J)	E_{cap} error (%)
50k	10	Alg. 1	6×2	1,159	49,507	-0.99
		ES	7×2	1,289	58,439	+16.88
50k	30	Alg. 1	12×1	415	50,970	+1.94
		ES	12×1	524	51,711	+3.42
100k	30	Alg. 1	12×2	648	100,011	+0.01
		ES	12×2	874	101,545	+1.55
200k	20	Alg. 1	10×5	2,691	197,713	-1.14
		ES	10×5	1713	201,702	+0.85

We confirm that the proposed MPTT design may have a slightly smaller E_{cap} than E_{req}. This is mainly because the curve on the P_{charge} surface is not an exact sine function. We can mitigate this error by a minor overdesign of E_{req}, which is anyway required due to the component tolerance in a real system.

6. CONCLUSIONS

This paper is the first paper that introduces a complete design and optimization framework for the maximum power point tracking (MPPT) of a solar cell and a supercapacitor system. While conventional MPPT maximizes the solar cell output power without considering power loss in the charger, we consider the charger loss and introduce a maximum power transfer tracking (MPTT) that explicitly maximizes the charging power that goes into the supercapacitor. Consideration of the charger efficiency, ranging from 10% to 80%, significantly affects the amount of harvested energy, and completely changes the design framework. We found that the capacitance of the supercapacitor bank and solar cell module output voltage are also important design parameters that alter the system efficiency, and formulated a joint optimization problem for the design parameters. Our proposed MPTT design framework derives the cost-optimal solar cell array configuration, the optimal supercapacitor bank capacitance, and keeps track of the true power-optimal points that are different from those of conventional MPPT. This framework significantly enhances the overall system efficiency from 3% to 6+ times. The MPTT framework can be implemented by the same method used for MPPT such as perturb & observe, incremental conduction, etc.

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