Accurate Component Model Based Optimal Control for Energy Storage Systems in Households with Photovoltaic Modules

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Abstract—**Integrating residential photovoltaic (PV) power generation and energy storage systems into the Smart Grid is an effective way of utilizing renewable power and reducing the consumption of fossil fuels. This has become a particularly interesting problem with the introduction of dynamic electricity energy pricing models since electricity consumers can use their PV-based energy generation and controllable energy storage devices for peak shaving on their power demand profile from the grid, and thereby, minimize their electricity bill cost. A realistic electricity price function is considered in this paper with billing period of a month, comprised of both the energy price component and the demand price component. Due to the characteristics of the realistic electricity price function and the energy storage capacity limitation, the residential storage control algorithm should properly account for various energy loss components during system operation, including the energy loss components due to rate capacity effect in the storage system as well as power dissipation in the power conversion circuitries. A near-optimal storage control algorithm is proposed accounting for these aspects, based on the PV power generation and load power consumption prediction results in the previous papers. The near-optimal control algorithm, which controls charging/discharging schemes of the storage system, is effectively implemented by solving a convex optimization problem at the beginning of each day with polynomial time complexity. Experimental results demonstrate that the proposed near-optimal residential storage control algorithm achieves up to 36.0% enhancement in electricity cost reduction than the baseline control algorithm.**

Keywords-Photovoltaic system; energy storage system; optimal control

I. INTRODUCTION

The traditional static and centralized structure of electricity grid consists of a transmission network, which transmits electrical power generated at remote power plants through long-distance high-voltage lines to substations, and a distribution network, which delivers electrical power from substations to local end users. In this infrastructure, the local distribution network is often statically adjusted to match the load profile from its end users. Since the end user profiles often change drastically according to the day of week and time of day, the Power Grid must be able to support the worst-case power demand of all end users in order to avoid potential power delivery failure (blackout or brownout) [1].

To avoid expending a large amount of capital in the future for expanding the power generation capacity to meet the expected growth of energy consumption among end users under the worst-case power demand conditions, progressive integration of smart meters and communication devices aims to transform the Power Grid to a decentralized Smart Grid, which can monitor and control the power flow in the Grid to match the amount of power generation to that of power consumption, and to minimize the overall cost of electrical energy delivered to the end users [2]. In the Smart Grid infrastructure, utility companies can deploy dynamic electricity pricing strategies incentivizing consumers to perform *demand side management* by adjusting their power demand from the Grid to match the power generation capacity of the Grid. There are several ways to perform such a demand side management, including integration of intermittent energy sources such as photovoltaic (PV) power or wind power at the residential level, demand shaping (i.e., consumers shift their tasks to the off peak periods), etc. [3]. In this paper we focus on the former solution, or more specifically, integrating PV power generation with the Smart Grid for residential usage.

Although integrating residential renewable energy sources into the Smart Grid is an effective way of utilizing renewable power and reducing the consumption of fossil fuels, several problems must be addressed for these benefits to be realized. First, there exists a mismatch between the peak PV power generation time (usually at noon) and the peak load power consumption time (usually in the evening) in each day. This timing skew results in conditions where the generated PV power cannot be optimally utilized to perform peak power shaving. One effective solution will be incorporating energy storage system into the PV assisted Smart Grid for residential users. The proposed residential energy storage system shall store power from the Smart Grid during off peak periods of each day and (or) from the PV system, and provide power for the end users during the peak periods of that day for peak power shaving (i.e., reducing the level of power that is taken from the Grid) and energy cost reduction (since electrical energy tends to be most expensive

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during these peak hours.) Therefore, the design of energy pricing-aware control algorithm for the residential storage system, which controls the charging and discharging of the storage system and the magnitude of charging/discharging current, is an important task in order for the Smart Grid technology to deliver on its promises.

A realistic electricity pricing function consists of both an *energy price* component, which is a time of usage (TOU) dependent function indicating the unit energy price during each time period of the billing period (a day, or a month, etc.), and a *demand price* component, which is an additional charge due to the peak power consumption in the billing period. The latter component is required in order to prevent a case whereby all customers utilize their PV power generation and energy storage systems and/or schedule their loads such that a very large amount of power is demanded from the Smart Grid during low-cost time slots, which can subsequently result in power delivery failure for the customers.

Moreover, the capacity of the storage system is limited due to the relatively high cost of electrical energy storage elements. Therefore, the following two requirements need to be satisfied so that the storage controller can perform optimization of the total cost induced by both the energy price and the demand price. First, it is important for the controller to forecast the PV power generation and load power consumption profiles during a billing period. Second, the storage control algorithm should accurately account for the energy loss in the storage charging/discharging process and in power conversion circuitries, in order to achieve optimality in total cost saving. This requirement implies taking into account accurate energy loss models for storage and power conversion circuitries in the controller's optimization framework. To satisfy the first requirement, references [4]-[6] are representatives of PV power generation and load power consumption profile prediction methods by either predicting the whole power profiles, or predicting certain statistical characteristics of these profiles. Our previous work [7] provides PV power generation and load power consumption profile prediction algorithms specifically designed for the residential Smart Grid controller. On the other hand, few research papers have focused on addressing the second requirement.

In this paper, we consider the case of a residential Smart Grid user equipped with local PV power generation and energy storage systems. We consider realistic electricity price function comprised of both energy and demand prices, with the billing period of a month [12]. We first provide the overall system architecture and the storage power loss model used in the paper. Based on the PV power generation and load power consumption prediction results from previous papers, we propose a near-optimal storage control algorithm accounting for the energy loss components due to power dissipation in the power conversion circuitries, as well as the *rate capacity effect*, which is the most significant portion of energy loss in the storage system especially when the storage system charging/discharging current is high. The proposed near-optimal storage control algorithm is effectively implemented by solving a convex optimization problem with polynomial time complexity at the beginning of each day in a billing period. Experimental results demonstrate that the proposed residential storage control algorithm achieves up to 36.0% enhancement in electricity cost reduction the baseline storage control algorithm.

The rest of this paper is organized as follows. We describe the system model, price function, and overall cost function in Section II. Section III presents the power loss model of the storage system. Section IV presents the residential storage control algorithm to minimize the total energy cost over a billing period. Experimental results and conclusion are presented in Section V and Section VI, respectively.

II. SYSTEM MODEL AND COST FUNCTION

In this paper, we consider an individual residential Smart Grid user equipped with PV power generation and energy storage systems, as shown in Figure 1. The PV system and the storage system are connected to the residential DC bus, via unidirectional and bidirectional DC-DC converters, respectively. The AC bus, which is further connected to the Smart Grid, is connected via an AC/DC interface (e.g., inverter, rectifier, and transformer circuitries) to the residential DC bus. The residential AC load (e.g., household appliances, lighting and heating equipments) is connected to the AC bus. In this paper, we consider the power loss in the above-mentioned power conversion circuitries.

We adopt a *slotted time* model, i.e., all system constraints as well as decisions are provided for discrete time intervals of equal and constant length. More specifically, each day is divided into T time slots, each of duration D. We use $T = 96$ and $D = 15$ minutes. We use set $\boldsymbol{S}$ to denote the set of all T time slots in each day.

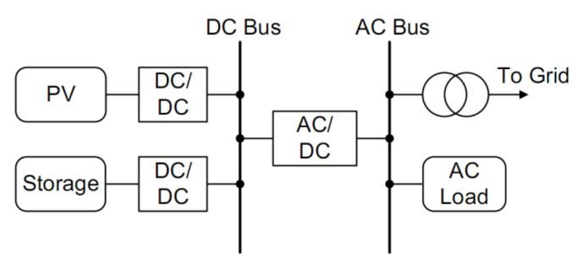

Figure 1: Block diagram showing the interface between PV module, storage system, residential load, and the Smart Grid.

We adopt a realistic electricity price function consisting of both the energy price component and the demand price component as discussed before, with a billing period of a month [12]. Consider a specific day i of a billing period. The residential load power consumption at the j^{th} time slot of that day is denoted by $P_{load,i}[j]$. The output power levels of PV and storages system at the j^{th} time slot are denoted by $P_{pv,i}[j]$ and $P_{st,i}[j]$, respectively, in which $P_{st,i}[j]$ can be positive (discharging from the storage), negative (charging the storage), or zero. We use $P_{grid,i}[j]$ to denote the power required from the Smart Grid, i.e., the *grid power*, at the j^{th} time slot of the i^{th} day, where $P_{grid,i}[j]$ can be positive (if the Smart Grid provides power for the residential usage), negative (if the residential system sells power back into the Grid), or zero. More accurately, such $P_{load,i}[j]$, $P_{pv,i}[j]$, $P_{st,i}[j]$, and $P_{grid,i}[j]$ values can be viewed as the average power generation or consumption values in the j^{th} time slot of the i^{th} day.

We consider realistic power conversion circuitries (i.e., their power conversion efficiency is less than 100%) in the proposed optimization framework. We use η_1, η_2 and η_3 to denote the power conversion efficiencies of the DC-DC converter connecting between the PV system and the DC bus, the DC-DC converter connecting between the storage system and the DC bus, and the AC/DC power conversion interface, respectively. Those power conversion efficiency values are typically in the range of 85% to 95%.

There are three *operating modes* in the system. In the first mode, both the PV system and the storage system are providing power for the residential load (i.e., the storage system is being discharged.) For the j^{th} time slot of the i^{th} day, the condition that the residential system is in the first mode is given by $P_{st,i}[j] \geq 0$. In this case, the grid power can be calculated by

$$P_{grid,i}[j] = P_{load,i}[j] - \eta_1 \cdot \eta_3 \cdot P_{pv,i}[j] \\ -\eta_2 \cdot \eta_3 \cdot P_{st,i}[j] \quad (1)$$

In the second mode, the storage system is being charged, and the PV system is sufficient for storage charging. For the j^{th} time slot of the i^{th} day, the condition that the residential system is in the second mode is given by $P_{st,i}[j] < 0$ and $\eta_1 P_{pv,i}[j] + \frac{1}{\eta_2} P_{st,i}[j] \geq 0$. In this mode, there is power flowing from the DC bus to the AC bus, and the grid power can be calculated by

$$P_{grid,i}[j] = P_{load,i}[j] - \eta_1\eta_3 \cdot P_{pv,i}[j] - \frac{\eta_3}{\eta_2} \cdot P_{st,i}[j] \quad (2)$$

In the third mode, the storage system is being charged, and the PV system is insufficient for charging the storage i.e., the storage is simultaneously charged by the PV system and the Grid. For the j^{th} time slot of the i^{th} day, the condition that the residential system is in the third mode is given by $P_{st,i}[j] < 0$ and $\eta_1 P_{pv,i}[j] + \frac{1}{\eta_2} P_{st,i}[j] < 0$. In this mode, there is power flowing from the AC bus to the DC bus, and the grid power is given by

$$P_{grid,i}[j] = P_{load,i}[j] - \frac{1}{\eta_3}\left(\eta_1 P_{pv,i}[j] + \frac{1}{\eta_2} P_{st,i}[j]\right) \quad (3)$$

It can be observed from (1) – (3) that $P_{grid,i}[j]$ is a piecewise linear (continuous) and monotonically decreasing function of $P_{st,i}[j]$, when $P_{pv,i}[j]$ and $P_{load,i}[j]$ values are given. $P_{grid,i}[j]$ is also a convex function of $P_{st,i}[j]$.

As specified in [12], the electricity price function is pre-announced by the utility company just before the start of each billing period, and it will not change until possibly the start of the next billing period. We use a general electricity price function as follows. We use $Price_E[j]$ to denote the unit energy price at the j^{th} time slot of a day. Then the cost we pay in a billing period due to energy price component is

$$Cost_E = \sum_{i=1}^{30} \sum_{j=1}^{96} Price_E[j] \cdot P_{grid,i}[j] \cdot D \quad (4)$$

The demand price component, on the other hand, is charged for the peak power drawn from the Grid over certain time periods in a billing period. A general definition of the demand price is given as follows. Let $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, ..., $\boldsymbol{S}_N$ be N different non-empty subsets of the original set of time slots $\boldsymbol{S}$, each corresponding to a specific time period, named by the term *price period*, in a day. A price period does not need to be continuous in time. For example, a price period can be from 10:00 to 12:59 and then from 17:00 to 19:59, as shown in [12]. Also those price periods in a day do no need to be mutually exclusive. We use $j \in \boldsymbol{S}_k$ to denote the statement that the j^{th} time slot in a day belongs to the k^{th} price period. Let $Price_{D,k}$ be the demand price charged over each k^{th} price period $\boldsymbol{S}_k$. Then the cost we have to pay in a billing period due to the demand price component is given by

$$Cost_D = \sum_{k=1}^{N} Price_{D,k} \cdot \max_{1 \leq i \leq 30, j \in \boldsymbol{S}_k} P_{grid,i}[j] \quad (5)$$

Obviously, the total cost for the residential user in a billing period (a month) is the sum of the two aforesaid cost components.

III. THE STORAGE MODEL

The most significant portion of power loss in the storage system, which is typically made of Lead-acid batteries or Li-ion batteries, is due to the rate capacity effect. To be more specific, high battery discharging current will reduce the amount of available energy that can be extracted from the battery, thereby reducing the battery life [8]. In other words, high-peak pulsed discharging current will deplete much more of the battery's stored energy than a smooth workload with the same total energy demand. We use *discharging efficiency* of a battery to denote the ratio of the battery's output current to the degradation rate of its stored charge. Then the rate capacity effect specifies the fact that the discharging efficiency of a battery decreases with the increase of the battery's discharging current. The rate capacity effect also affects the energy loss in the battery during the charging process in a similar way.

The rate capacity effect can be captured using the Peukert's formula, an empirical formula specifying the battery charging and discharging efficiencies as functions of the charging current I_c and the discharging current I_d, respectively:

$$\eta_{rate,c}(I_c) = \frac{1}{(I_c/I_{ref})^{\alpha_c}}, \quad \eta_{rate,d}(I_d) = \frac{1}{(I_d/I_{ref})^{\alpha_d}} \qquad (6)$$

where α_c and α_d are Peukert's coefficients, and their values are typically in the range of 0.1 – 0.3; I_{ref} denotes the *reference current* of the battery, which is proportional to the battery's nominal capacity C_{nom}.

We name I_c/I_{ref} and I_d/I_{ref} the battery's *normalized charging current* and *normalized discharging current*, respectively. One can notice that the efficiency values $\eta_{rate,c}(I_c)$ and $\eta_{rate,d}(I_d)$ in Eqn. (6) are greater than 100% if the magnitude of the normalized charging or discharging current is less than one, which implies that the above-mentioned Peukert's formula is not accurate in this case. We modify the Peukert's formula such that the efficiency values $\eta_{rate,c}(I_c)$ and $\eta_{rate,d}(I_d)$ become equal to 100% if the magnitude of the normalized charging/discharging current is less than one. In other words, the battery suffers from no rate capacity effect in this case.

We denote the increase/degradation rate of storage energy in the j^{th} time slot of the i^{th} day by $P_{st,in,i}[j]$, which may be positive (discharging from the storage, and the amount of energy decreases), negative (charging the storage, and the amount of energy increases) or zero. Based on the modified Peukert's formula, the relation between $P_{st,in,i}[j]$ and the storage output power $P_{st,i}[j]$ is characterized by

$$P_{st,i}[j] = \begin{cases} V_{st} \cdot I_{st,ref} \left(\frac{P_{st,in,i}[j]}{V_{st} \cdot I_{st,ref}}\right)^{\beta_1}, \text{if } \frac{P_{st,in,i}[j]}{V_{st} \cdot I_{st,ref}} > 1 \\ P_{st,in,i}[j], \text{ if } -1 \le \frac{P_{st,in,i}[j]}{V_{st} \cdot I_{st,ref}} \le 1 \\ -V_{st} \cdot I_{st,ref} \left(\frac{|P_{st,in,i}[j]|}{V_{st} \cdot I_{st,ref}}\right)^{\beta_2}, \text{if } \frac{P_{st,in,i}[j]}{V_{st} \cdot I_{st,ref}} < -1 \end{cases} \qquad (7)$$

where V_{st} is the storage terminal voltage and is supposed to be (near-) constant; $I_{st,ref}$ is the reference current of the storage system, which is proportional to its nominal capacity $C_{st,nom}$ given in Ahr; coefficient β_1 is in the range of 0.8 – 0.9, and coefficient β_2 is in the range of 1.1 – 1.3.

One can observe that when the storage discharging (or charging) current is the same, the discharging (or charging) efficiency becomes higher (i.e., the rate capacity effect becomes less significant) when the nominal capacity of the storage system is larger.

We use function $P_{st,i}[j] = f_{st}(P_{st,in,i}[j])$ to denote the relation between $P_{st,i}[j]$ and $P_{st,in,i}[j]$. One important observation is that such function is a concave and monotonically increasing function over the input domain $-\infty < P_{st,in,i}[j] < \infty$, as shown in Figure 2. Due to the monotonicity property, $P_{st,in,i}[j]$ is also a monotonically increasing function of $P_{st,i}[j]$, denoted by $P_{st,in,i}[j] = f_{st}^{-1}(P_{st,i}[j])$. We can see from Figure 2 that Lead-acid battery-based storage system has more significant energy loss due to rate capacity effect than Li-ion battery-based storage system. However, Lead-acid battery-based storage system is more often deployed in real scenarios due to cost considerations.

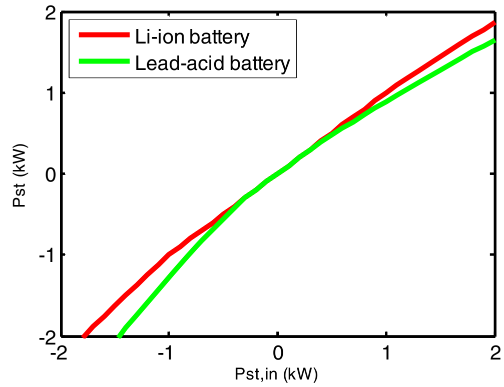

Figure 2: Relationship between $P_{st,i}[j]$ and $P_{st,in,i}[j]$ in two types of batteries.

IV. OPTIMAL CONTROL OF RESIDENTIAL STORAGE

In this section, we introduce in detail the proposed near-optimal residential storage control algorithm, which could effectively utilize the combination of PV power generation and load power consumption prediction results to minimize the total electricity cost, including both the energy price and the demand price, over each billing period (a month.) The storage control optimization problem is performed at time 00:00 (i.e., at the beginning) of each day in the billing period. To be more realistic, we assume that the prediction results of PV power generation and load power consumption profiles at each i^{th} day are not available before time 00:00 of that day. We assume that the PV power generation and load power consumption profiles at each day, i.e., $P_{pv,i}[j]$ and $P_{load,i}[j]$ for $1 \le j \le 96$, can be accurately predicted from the weather forecast and prediction algorithms presented in our previous work [7]. At time 00:00 of each i^{th} day, the storage controller performs optimization to find the optimal storage system output power profile $P_{st,i}[j]$ for $1 \le j \le 96$ throughout the day, which is equivalent to finding the charging/discharging current profile of the storage system.

In this section, we first introduce the storage control optimization performed at the beginning of a billing period (i.e., at time 00:00 of day $i = 1$), in order to achieve a balance between the expected $Cost_E$ (induced by the energy price component) and $Cost_D$ (induced by the demand price component) values. Next, we introduce the storage control optimization performed at the beginning of the subsequent days in the billing period. Although in reality we control the output power $P_{st,i}[j]$ ($1 \le i \le 30$, $1 \le j \le 96$) of the storage system during system operation, we use $P_{st,in,i}[j]$ ($1 \le i \le 30$, $1 \le j \le 96$) as the control variables in the optimal storage control problem formulation since it can help transform the optimal storage control problem into a standard convex optimization problem. We observe from Eqns. (1), (2), (3), and (7) that the grid power $P_{grid,i}[j]$ ($1 \le i \le 30$, $1 \le j \le 96$) is a monotonically decreasing function of $P_{st,in,i}[j]$, denoted by $P_{grid,i}[j] = f_{grid}(P_{st,in,i}[j])$, over the input domain $-\infty < P_{st,in,i}[j] < \infty$. Again we have $P_{st,in,i}[j] = f_{grid}^{-1}(P_{grid,i}[j])$. Furthermore, $P_{grid,i}[j] = f_{grid}(P_{st,in,i}[j])$ is a convex function of the control variable $P_{st,in,i}[j]$ according to the rules of convexity in function composition [13], due to the following reasons: (i) $P_{grid,i}[j]$ is a convex and monotonically decreasing function of $P_{st,i}[j]$, and (ii) $P_{st,i}[j] = f_{st}(P_{st,in,i}[j])$ is a concave function of $P_{st,in,i}[j]$.

At any time in a billing period, let $Peak_k$ ($1 \le k \le N$) denote the peak grid power consumption value *observed so far* over the k^{th} price period in the billing period of interest. Obviously, such $Peak_k$ values are initialized to zero at the beginning of the billing period, and are updated at the end of each day.

A. Storage Control Optimization at the Beginning of a Billing Period

We introduce the storage control optimization at the beginning of a billing period (i.e., at time 00:00 of day $i = 1$), in order to achieve a balance between the expected $Cost_E$ and $Cost_D$ values. At that time, we have $Peak_k = 0$ for $1 \le k \le N$. The storage controller is only aware of the PV power generation and load power consumption profiles in the 1st day of the billing period, i.e., $P_{pv,1}[j]$ and $P_{load,1}[j]$ for $1 \le j \le 96$. The storage controller finds the optimal $P_{st,in,1}[j]$ profile for $1 \le j \le 96$. The objective of the storage controller is to minimize the estimated total electricity cost in the billing period. Then the Optimal Storage Control problem performed at the Beginning of a billing period (the OSC-B problem) is formally described as follows:

The OSC-B Problem Formulation

Given the PV power generation and load power consumption profiles of the 1st day in the billing period, i.e., $P_{pv,1}[j]$ and $P_{load,1}[j]$ for $1 \le j \le 96$, and the initial storage energy $E_{st,ini,1}$ at time 00:00.

Find the optimal $P_{st,in,1}[j]$ profile for $1 \le j \le 96$.

Minimize the estimated total electricity cost in the billing period, given by:

$$\begin{aligned} &Cost_D + Cost_E \\ &= 30 \cdot \sum_{j=1}^{96} Price_E[j] \cdot P_{grid,1}[j] \cdot D \\ &+ \sum_{k=1}^{N} Price_{D,k} \cdot \max\left(Peak_k, \max_{j \in \boldsymbol{S}_k} P_{grid,1}[j]\right) \end{aligned} \tag{8}$$

Subject to the following constraints:

For each $1 \le j \le 96$:

$$-P_{MAX,c} \le P_{st,in,1}[j] \le P_{MAX,d} \tag{9}$$

$$0 \le E_{st,ini,1} - \sum_{l=1}^{j} P_{st,in,1}[l] \cdot D \le E_{st,full} \tag{10}$$

$$E_{st,ini,1} - \sum_{j=1}^{96} P_{st,in,1}[j] \cdot D \ge E_{st,ini,1} \tag{11}$$

In the OSC-B problem formulation, the objective function (8) is the estimated total electricity cost in the billing period, since the storage controller is only aware of the PV power generation and load power consumption profiles in the 1st day, i.e., $P_{pv,1}[j]$ and $P_{load,1}[j]$ for $1 \le j \le 96$. Constraint (9) represents the restrictions on the maximum allowable amount of power flowing into and out of the storage system during charging and discharging, respectively. Constraint (10) ensures that the storage energy can never become less than zero or exceed a maximum value $E_{st,full}$ throughout the day. Constraint (11) ensures that the remaining storage energy at the end of day is no less than the initial value $E_{st,ini,1}$. This amount of energy is required for performing peak power shaving in the next day.

The OSC-B problem is a standard convex optimization problem due to the following reasons:

- The objective function (8) is a convex objective function since the pointwise maximum function of a set of convex functions is still a convex function.
- The other constraints are all convex (or linear) inequality constraints.

Although the OSC-B problem is formulated as a convex optimization problem and therefore it can be solved optimally in polynomial time complexity using convex optimization algorithms [13], it is hard to directly solve the OSC-B problem using standard convex optimization tools such as CVX [14] or the *fmincon* function in MATLAB. This is because the function $P_{grid,i}[j] = f_{grid}(P_{st,in,i}[j])$ is non-differentiable at several points, and typical convex optimization tools only accept differentiable objective functions. Therefore, we use a piecewise linear function to approximate the function $f_{grid}(P_{st,in,i}[j])$, and then transform the OSC-B problem into a linear programming problem (please note that all the constraints are linear constraints), which could be optimally solved using standard optimization tools in polynomial time complexity. Details are omitted due to space limitation. Similar method will also be applied to the optimal storage control problem as shall be discussed in Section IV.B.

B. Storage Control Optimization at the Beginning of the Subsequent Days

We introduce the storage control optimization at the beginning of the subsequent days of the billing period. Suppose that we are at the beginning of the i^{th} day of the billing period of interest. At that time, the $Peak_k$ values may not be zero any more. The storage controller is aware of the PV power generation and load power consumption profiles in the i^{th} day, i.e., $P_{pv,i}[j]$ and $P_{load,i}[j]$ for $1 \leq j \leq 96$. The storage controller finds the optimal $P_{st,in,i}[j]$ profile for $1 \leq j \leq 96$. The objective of the storage controller is to minimize *the increase of the electricity cost* in the i^{th} day of the billing period, as shall be formally described in the following. The Optimal Storage Control problem performed at the beginning of the Subsequent days in the billing period (the OSC-S problem) is formally described as follows:

The OSC-S Problem Formulation

Given the PV power generation and load power consumption profiles of the i^{th} day in the billing period, i.e., $P_{pv,i}[j]$ and $P_{load,i}[j]$ for $1 \leq j \leq 96$, and the initial storage energy $E_{st,ini,i}$ at time 00:00.

Find the optimal $P_{st,in,i}[j]$ profile for $1 \leq j \leq 96$.

Minimize the increase in the electricity cost in the i^{th} day, given by:

$$\sum_{j=1}^{96} Price_E[j] \cdot P_{grid,i}[j] \cdot D + \sum_{k=1}^{N} Price_{D,k} \left(\max\left(Peak_k, \max_{j \in S_k} P_{grid,i}[j] \right) - Peak_k \right) \quad (12)$$

or equivalently, minimize:

$$\sum_{j=1}^{96} Price_E[j] \cdot P_{grid,i}[j] \cdot D + \sum_{k=1}^{N} Price_{D,k} \cdot \max\left(Peak_k, \max_{j \in S_k} P_{grid,i}[j] \right) \quad (13)$$

Subject to the following constraints:

For each $1 \leq j \leq 96$:

$$-P_{MAX,c} \leq P_{st,in,i}[j] \leq P_{MAX,d} \quad (14)$$

$$0 \leq E_{st,ini,i} - \sum_{l=1}^{j} P_{st,in,i}[l] \cdot D \leq E_{st,full} \quad (15)$$

$$E_{st,ini,i} - \sum_{j=1}^{96} P_{st,in,i}[j] \cdot D \geq E_{st,ini,i} \quad (16)$$

In the OSC-S problem formulation, the objective function (12) is the increase of the electricity cost in the i^{th} day of the billing period of interest. The objective function consists of two parts: (i) the energy price-induced electricity cost in the i^{th} day of the billing period, given by the first term of Eqn. (12), and (ii) the increase in the demand price-induced electricity cost in the billing period of interest, given by the second term of Eqn. (12).

The constraints of the OSC-S problem are similar to the constraints in the OSC-B problem discussed in Section IV.A. Again, similar to the OSC-B problem, the OSC-S problem has convex objective function (12) and linear inequality constraints, and therefore, it can be solved optimally in polynomial time complexity using standard convex optimization methods [13][14].

V. Experimental Results

In this section, we present the experimental results on the effectiveness of the proposed accurate component model-based near-optimal residential storage control algorithm. The PV power profiles used in our experiments are measured at Duffield, VA, in the year 2007, while the electric load data comes from the Baltimore Gas and Electric Company, also measured in the year 2007.

We use the electricity price function similar to [7][12], given as follows. The energy price component is given by: 0.01879 \$/kWh during 00:00 to 09:59 and 20:00 to 23:59, 0.03952 \$/kWh during 10:00 to 12:59 and 17:00 to 19:59, 0.04679 \$/kWh during 13:00 to 16:59. For the monthly demand price component, there are three price periods in a day: (i) the "high peak" period from 13:00 to 16:59, with demand price of 9.00 \$/kW, (ii) the "low peak" period from 10:00 to 12:59 and from 17:00 to 19:59, with demand price of 3.25 \$/kW, (iii) the "overall" period from 00:00 to 23:59 (the whole day), with demand price of 5.00 \$/kW.

We define the *cost saving capability* of a storage control algorithm (the proposed algorithm or the baseline algorithm) to be the average daily cost saving over a billing period due to the additional storage system, compared with the same residential Smart Grid user equipped only with the PV system. We compare the cost saving capabilities of our proposed near-optimal storage control algorithm with the baseline algorithm. The baseline algorithm charges the storage system from the Grid during the "off peak" period (00:00 to 09:59 and 20:00 to 23:59) with constant power, and distributes energy stored in the storage system evenly in the "high peak" period.

Table 1 shows the comparison results on the cost saving capabilities between our proposed near-optimal storage control algorithm and the baseline algorithm on every month throughout a year, when the capacity of the storage system is 45 Ah. The improvement in cost saving capabilities using the proposed algorithm is provided in the table. Table 2 shows the comparison results on the same testing data when the capacity of the storage system is 60 Ah. We can see from the two tables that the proposed near-optimal residential storage control algorithm consistently outperforms the baseline algorithm, with a maximum cost saving capability improvement of 36.0% (on December, 60 Ah storage capacity.) Furthermore, it can be observed that the proposed storage control algorithm demonstrates more significant cost saving capability gain over the baseline systems when the storage system has a capacity of 60 Ah. It also has higher cost saving capability during the winter than during the summer. The reason is that the peak load power consumption generally occurs in the "high peak" price period in the summer, and therefore, the baseline algorithm achieves relatively better performance by distributing the storage energy in the "high peak" periods.

TABLE I. IMPROVEMENT IN COST SAVING CAPABILITY OF THE PROPOSED ALGORITHM COMPARED WITH THE BASELINE ALGORITHM, WHEN THE STORAGE CAPACITY IS 45 AH.

Month	Jan.	Feb.	Mar.	Apr.
Improvement	26.3%	22.6%	12.4%	12.8%
Month	May	Jun.	Jul.	Aug.
Improvement	10.3%	10.2%	10.6%	10.4%
Month	Sep.	Oct.	Nov.	Dec.
Improvement	12.0%	15.3%	17.3%	27.2%

TABLE II. IMPROVEMENT IN COST SAVING CAPABILITY OF THE PROPOSED ALGORITHM COMPARED WITH THE BASELINE ALGORITHM, WHEN THE STORAGE CAPACITY IS 60 AH.

Month	Jan.	Feb.	Mar.	Apr.
Improvement	34.6%	30.0%	16.5%	17.1%
Month	May	Jun.	Jul.	Aug.
Improvement	13.4%	13.1%	14.0%	13.7%
Month	Sep.	Oct.	Nov.	Dec.
Improvement	14.9%	20.1%	23.0%	36.0%

VI. CONCLUSION

This paper addresses the problem of integrating residential PV power generation and storage systems into the smart grid for simultaneous peak power shaving and total electricity cost minimization over a billing period, making use of the dynamic energy pricing models. The residential storage control algorithm should properly account for the energy loss in storage charging/discharging and in power conversion circuitries. Based on the PV power generation and load power consumption prediction methods in our previous papers, we propose the accurate component model-based near-optimal storage control algorithm accounting for this aspect. The near-optimal storage control algorithm is effectively implemented by solving a convex optimization problem at the beginning of each day with polynomial time complexity. Experimental results demonstrate that the proposed residential storage control algorithm achieves up to 36.0% more total electricity cost reduction compared with the baseline control algorithm.

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