# An Electricity Trade Model for Multiple Power Distribution Networks in Smart Energy Systems

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***Abstract*—The future smart energy systems are projected to be decentralized power networks, each consisting of various types of renewable power generators that serve a small group of energy users. Interaction between different power networks through energy trading over a marketplace provides the chance to fully utilize the capacity of each power generator type. As a result of this interaction, the power generation and distribution levels can be decided for each time slot in order to achieve a maximal utility. In this paper, an electricity trade model is introduced for decentralized power networks to deal with the utility maximization problem. In the proposed model, multiple power networks can trade among each other and thus each of them can achieve a utility increase from making use of its comparative advantage on power generation during a certain period of time. The model is studied from several special scenarios to a more general scenario and an efficient solution is presented for each scenario. Experimental result validates the accuracy and efficiency of the presented solutions.**

## I. INTRODUCTION

Energy efficiency optimization has drawn significant attention especially with the increase of energy prices, the rise of energy usage and the deteriorating environment [1], [2]. Smart energy systems are thus introduced which aim at increasing the efficiency of both energy generation and distribution. The state-of-the-art smart energy systems, including resource allocation frameworks in cloud computing, smart grid infrastructures, and electric mobility systems, are undergoing a transformation from a centralized, producer-controlled network to one that is decentralized and consumer-interactive [1]-[3]. In order to switch from a centralized power distribution network (in which electrical energy is generated by a few far-off high-capacity generators and transferred to local end-users) to a decentralized network infrastructure, it is necessary to implement small-scale generators that are located closely to the points-of-use. This distributed power generation scheme makes it easier to incorporate and utilize all kinds of renewable energy sources, and can significantly reduce the energy transmission cost [3].

This however makes it challenging to operate and control the decentralized power network with increasing number of distributed energy sources. Authors of [4] introduced the notion of *microgrid* to tackle this problem. A microgrid is a small scale power network, which contains one or multiple types of renewable power generators and energy storages serving a single or a small group of energy users, and offers the possibility of coordinating the distributed resources in a more intelligent way so that they can behave as a controlled entity and achieve energy autonomy without relying on external power sources.

For each power network, it is necessary to match power demand with supply as response to the intermittency of renewable power sources such as wind and solar [5]. This however is very challenging due to the fact that power demand depends on a variety of factors of energy users and also energy users may impose their preference of energy usage at certain times. It is also worth noting that various types of power generation centers experience different power generation capabilities and costs as a function of time and weather conditions, e.g., a photovoltaic power generator cannot generate power during night time. The problem of matching demand with supply becomes even more complicated with the adoption of energy storage [5]. For a power network, the amount of energy generation and consumption needs to be decided under certain resource constraints and with the goal of maximizing the consumer satisfaction level [6].

The roadmaps of state-of-the-art microgrid power network structures are mainly based on the marketing ideas [7]. Compared with adopting energy storage, it is more cost-efficient for one power network to sell its surplus power generation to meet the demand of its neighborhoods, with the exchange of purchasing energy from other power networks during its energy shortage times. By trading with other networks, each power network can make use of its comparative advantage and achieve an increase of user satisfaction. Previous work such as [7] has analyzed the structure of the energy trading platform involving the microgrid neighborhood. However, they lack a thorough analysis of economic factors such as the best cooperative decision on energy generation, the optimal trading volume of energy, and also the determination of relative energy prices.

To provide a detailed study on the above problems, we present an electricity trade model for power networks in this paper. In this model, multiple power networks can trade among each other and as a consequence, each one of them can achieve an increase in the user satisfaction level from making use of its comparative advantage on power generation during a certain period of time. We elaborate the proposed model at some special scenarios and then at a general scenario. We provide optimal solutions for both the special scenarios and the general scenario, in which the idea of comparative advantage is employed to significantly reduce the problem complexity.

The remainder of this paper is organized as follows. In the next section, we present the system model for electricity trading among multiple power networks to maximizes the user satisfaction level. In Section III, we discuss the presented model in detail from several special scenarios to a more general scenario. Section IV reports experimental results and the paper is concluded in Section V.

## II. SYSTEM MODEL

In our model, there are $N$ power networks in the smart energy system, and each of them contains one or several types of power generators serving a small group of energy users. A slotted time model is assumed, i.e., all system cost parameters and constraints as well as energy generation and consumption decisions are provided for discrete time intervals of constant length. For example, if we set the *operating period* to be a day and $T$ = 6, a day is divided into 6 time slots, each with duration of four hours. We use $P_{n,i}$ and $C_{n,i}$ to represent the energy generation and consumption levels, respectively, for each power network $n$ at time slot $i$ where $n \in \{1,2,\dots,N\}$ and $i \in \{1,2,\dots,T\}$.

In the perspective of economists, users consume commonalities (such as energy) at each time because this energy consumption provides satisfaction for users. The level of satisfaction is represented by *utility* [8]. We use the famous Cobb-douglas utility function, which models that for each power network $n$, the relationship between the utility and the level of energy consumption at each time follows the form [8]:

$$U_n = \prod_{i=1}^{T} C_{n,i}^{\alpha_i}, \tag{1}$$

where $\alpha_i$ is the *preference factor* at each time slot and we have $0 < \alpha_i < 1$ for all $i$. A higher $\alpha_i$ means that the users prefer to consume more energy at the corresponding time slot. Although an individual might have his own energy usage preference, it can be assumed that different power networks share the same set of community preference factors because each power network is connected to a number of users, which means at any time slot $i$, $\alpha_i$ is the same for different power network $n$.

Basically, an electricity trade contract consists of two steps: First, all the power networks get together to decide the total energy generation at each time slot in order to achieve a maximal overall utility function; Second, the power networks "bargain" among each other to decide the distribution of the total energy generation in a fair way. In the following subsections, we will present the system model for each step.

## A. Model for Cooperative Energy Generation

In international economics studies, countries are willing to engage in international trades because they are distinct from each other and all of them can benefit from the differences by reaching an arrangement in which each country does the things it does relatively well [8]. Similarly, in the interaction of various power networks, each power network can perform electricity trade with its neighbors in order to achieve a utility increase.

In this model, we assume that the power networks have different energy generation costs as well as total resources. For each power network $n$, we denote the total resource by $I_n$, and the energy generation at each time slot by $P_{n,i}$. It is also commonly modeled in economics that under a given resource constraint, the *production possibilities* of energy generation should follow the given equation [8]:

$$\sum_{i=1}^{T} \beta_{n,i} \cdot P_{n,i} = I_n, \tag{2}$$

where for each network $n$, $\beta_{n,i}$ is the number of resource units that are required to generate one unit of energy at each time slot and $I_n$ is the total number of resource units that are allowed to use during one operating period. Notice that $\beta_{n,i}$ is determined by the type of energy generators as well as the level of technology, and might not be constant for different $i$ values. If a network cannot generate energy at all during certain time slots (e.g. photovoltaic generators cannot generate energy during night time), then $\beta_{n,i}$ is set to $+\infty$.

We aim at achieving a global maximum utility, so $C_i$ is used to denote the total energy consumption (equal to the total energy generation) at time slot $i$. In addition, as stated before, $\alpha_i$ denotes the corresponding preference factor, which has been assumed to be the same for all power networks. Using the above definitions, the cooperative energy generation problem can be modeled as follows:

**Cooperative Energy Generation Problem to Maximize the Total Utility**

***Find*** the optimal energy generation $P_{n,i}$ for $1 \le n \le N$, $1 \le i \le T$.
***Maximize:***

$$U = \prod_{i=1}^{T} C_i^{\alpha_i}$$

***Subject to:***

$$\sum_{i=1}^{T} \beta_{n,i} \cdot P_{n,i} = I_n, \qquad \forall\, 1 \le n \le N$$

$$P_{n,i} \ge 0, \qquad \forall\, 1 \le n \le N, 1 \le i \le T$$

$$C_i = \sum_{n=1}^{N} P_{n,i} = \sum_{n=1}^{N} C_{n,i}, \qquad \forall\, 1 \le i \le T$$

## B. Model for Energy Distribution

As the total energy generation at each time slot is determined, the power networks need to make decisions on energy distribution, i.e., determining the energy consumption level at each power network. The rule of energy distribution should be based on the contribution made by each of the power networks.

In our model, the energy distribution is performed using a method that we refer to as the *fair benefit distribution law* that results in the same *utility increase ratio* (the ratio of the utility function after trade to the maximal utility function before trade, i.e., $U_{n,trade}/U_{n,local}$) for every power network $n$. With the same set of preference factors, it can be easily proven that the most efficient distribution method that maximizes the utility increase ratio (which is the same for all power networks after trade) can be performed as follows: for $1 \le n \le N$, $1 \le i \le T$, we have

$$C_{n,i} = \frac{U_{n,local}}{\sum_{j=1}^{N} U_{j,local}} \cdot C_i, \tag{3}$$

where $C_i$ is the total generated energy at time slot $i$, and $U_{n,local}$ can be determined using the local maximization method that will be discussed in the next section.

The problem of energy distribution is relatively simple, and hence, we do not discuss in detail in this paper because of space limit (but it is used and shown in experimental result section). In the next section, we focus on the cooperative energy generation problem. Several special scenarios are studied and finally a general solution is provided.

# III. Utility Maximization Solutions

In this section, different scenarios are studied for the above-mentioned cooperative energy generation problem. We differentiate these scenarios based on the number of power networks $N$ and the total number of time slots $T$.

## A. $N = 1$:

When $N = 1$, there is only one power network. This case is considered as a closed economy group in terms of energy (i.e., no energy trading is allowed) and decides the optimal energy generation distribution at different times. In this case, we have $C_i = P_{1,i}$ for $1 \le i \le T$. The convex optimization technique [9] can be used to numerically solve this problem.

**Property 1:** At the optimal solution point of the above local utility/welfare maximization problem, we have:

$$\frac{\partial U/\partial C_i}{\partial I_1/\partial P_{1,i}} = \frac{\alpha_i \cdot U}{\beta_{1,i} \cdot C_i} = D \quad \forall\, 1 \le i \le T, \tag{4}$$

where $D$ is the same value for all $i$, which results in the optimal solution given by:

$$P_{1,i} = C_i = \frac{\alpha_i}{\beta_{1,i} \cdot \sum_{m=1}^{T} \alpha_m} \cdot I_1 \quad \forall\, 1 \le i \le T. \tag{5}$$

***Proof:*** Assume at the optimal solution point, there exists a pair of time slots $\{i, j\}$ with $\frac{\partial U/\partial C_i}{\partial I_1/\partial P_{1,i}} > \frac{\partial U/\partial C_j}{\partial I_1/\partial P_{1,j}}$. We will be able to find a new feasible solution with $C_i' = C_i + \sigma/\beta_{1,i}$ and $C_j' = C_j - \sigma/\beta_{1,j}$, where $\sigma$ is a very small value with the same unit of total resource $I_1$. As we have $\frac{\partial U/\partial C_i}{\partial I_1/\partial P_{1,i}} > \frac{\partial U/\partial C_j}{\partial I_1/\partial P_{1,j}}$, it can be proven that $U(C_1, C_2, \dots, C_i', C_j', \dots) > U(C_1, C_2, \dots, C_i, C_j, \dots)$, which contradicts the optimality of the solution. As a result, the optimal solution will occur only when $\frac{\partial U/\partial C_i}{\partial I_1/\partial P_{1,i}}$ is the same for every $i$. ■

The $N = 1$ scenario is relatively simple. However, the solution of the simple scenario is necessary to determine the general solution for more complex scenarios.

### B. $N = 2, T = 2$:

In the simplest electricity trading scenario, there are two power networks and an operating period is divided into only two time slots. Economists have used the notion of *comparative advantage* to analyze the motive of trading. Comparative advantage refers to the ability of a entity to produce a particularly good or service at a lower marginal cost or opportunity cost over another [8]. For energy generation in a power network, the comparative advantage comes from the ability of generating energy at a particular time slot at a lower opportunity cost over the energy generators in another power network. For convenience, assume we have re-labeled the time slots to make $\frac{\beta_{1,1}}{\beta_{2,1}} < \frac{\beta_{1,2}}{\beta_{2,2}}$. Based on the definition, the first power network has comparative advantage on energy generation at time slot 1, and the second has comparative advantage on energy generation at time slot 2. Notice that comparative advantage is determined by the ratio of $\frac{\beta_{1,i}}{\beta_{2,i}}$ instead of the absolute values, which means that even though a power network has a higher energy generation cost at every time slot, it can still have comparative advantage at some of the time slots. Electricity trading aims at enabling both power networks to make better use of its comparative advantage at a special time slot, and the optimal solution can be determined accordingly.

**Property 2:** At the optimal solution point of the utility maximization problem with $N = 2$ and $T = 2$, if $\frac{\beta_{1,1}}{\beta_{2,1}} < \frac{\beta_{1,2}}{\beta_{2,2}}$, we have either $P_{2,1} = 0$ or $P_{1,2} = 0$ (or both).

***Proof:*** Assume we have both $P_{2,1} > 0$ or $P_{1,2} > 0$ at the optimal solution point, we will be able to find a new feasible solution with $P_{1,1}' = P_{1,1} + \sigma_1/\beta_{1,1}$, $P_{1,2}' = P_{1,2} - \sigma_1/\beta_{1,2}$, $P_{2,1}' = P_{2,1} - \sigma_2/\beta_{2,1}$ and $P_{2,2}' = P_{2,2} + \sigma_2/\beta_{2,2}$, where $\sigma_1$ and $\sigma_2$ are very small values with the same unit of total resource $I_1$ and $I_2$. Since $\frac{\beta_{1,1}}{\beta_{2,1}} < \frac{\beta_{1,2}}{\beta_{2,2}}$, there should exist a pair of $\{\sigma_1, \sigma_2\}$ such that $\frac{\beta_{1,1}}{\beta_{2,1}} < \frac{\sigma_1}{\sigma_2} < \frac{\beta_{1,2}}{\beta_{2,2}}$. In this case, we will have both $P_{1,1}' + P_{2,1}' > P_{1,1} + P_{2,1}$ and $P_{1,2}' + P_{2,2}' > P_{1,2} + P_{2,2}$, which leads to a higher total utility and contradicts the optimality of the solution. ■

Based on this property, at least one power network will be assigned to generate energy at one time slot at the optimal solution point. And thus the optimal solution of the utility maximization problem with $N = 2$ and $T = 2$ can be achieved by comparing the optimal solutions with $P_{2,1} = 0$ or $P_{1,2} = 0$.

Notice that there is another special situation where $\frac{\beta_{1,1}}{\beta_{2,1}} = \frac{\beta_{1,2}}{\beta_{2,2}}$. It can be proven that there are multiple optimal solution points in this case and one of the optimal solution can be achieved when each of the two power networks simply maximizes its own utility as in the case of $N = 1$, which means that none of the two power networks can gain from trading. We will not discuss this case in detail because of space limitation but it is validated in the experimental results section.

### C. $N > 2, T = 2$:

The above scenario can be extended to a multiple-network- two-slot situation. Assume we have already re-labeled the time slots so that we have $\frac{\beta_{1,1}}{\beta_{1,2}} < \frac{\beta_{2,1}}{\beta_{2,2}} < \frac{\beta_{3,1}}{\beta_{3,2}} < \dots < \frac{\beta_{N,1}}{\beta_{N,2}}$. We come up with the property as follows:

**Property 3:** At the optimal solution point of the utility maximization problem with $N > 2$ and $T = 2$, if $\frac{\beta_{1,1}}{\beta_{1,2}} < \frac{\beta_{2,1}}{\beta_{2,2}} < \frac{\beta_{3,1}}{\beta_{3,2}} < \dots < \frac{\beta_{N,1}}{\beta_{N,2}}$, there exists a number $n^*$ such that we have $P_{n,2} = 0$ for $\forall 1 \le n < n^*$ and $P_{n,1} = 0$ for $\forall n^* < n \le N$. In other words, there exists at most one (probably zero) power network that will generate energy at both time slots.

***Proof:*** When there already exists a number $n^*$ with both $P_{n^*,1} = 0$ and $P_{n^*,2} = 0$, if there exists another number $1 \le n < n^*$ with $P_{n,2} > 0$, we will have $\frac{\beta_{n,1}}{\beta_{n,2}} < \frac{\beta_{n^*,1}}{\beta_{n^*,2}}$ (equivalently, $\frac{\beta_{n,1}}{\beta_{n^*,1}} < \frac{\beta_{n,2}}{\beta_{n^*,2}}$) together with $P_{n,2} > 0$ and $P_{n^*,1} > 0$. We can find a better solution according to the proof of Property 2. Similarly, there should not exist another number of $n^* < n \le N$ with $P_{n,1} > 0$. ■

Based on the above property, when $n^*$ is given, $N - 1$ power networks are specified in energy generation at only one time slot and $P_{n,1}$ or $P_{n,2}$ can be simply determined based on the total resource $I_n$ for $n \ne n^*$, and it is easy to solve the utility maximization problem with only two variables $P_{n^*,1}$ and $P_{n^*,2}$. The remaining problem is to determine the optimal value of $n^*$, and a straightforward algorithm is to enumerate $n^*$ from 1 to *N*. However, we can easily prove that the total utility is a unimodal concave function with respect to $C_1$ or $C_2$, a more efficient method can be found. The detailed algorithm is omitted because of space limitation.

### D. $N = 2, T > 2$:

The two-network-multiple-slot case is similar to the previous one. Assume we have already re-labeled the time slots so that $\frac{\beta_{1,1}}{\beta_{2,1}} < \frac{\beta_{1,2}}{\beta_{2,2}} < \frac{\beta_{1,3}}{\beta_{2,3}} < \dots < \frac{\beta_{1,T}}{\beta_{2,T}}$. We come up with the similar property as follows:

**Property 4:** At the optimal solution point of the total utility/welfare maximization problem with $N = 2$ and $T > 2$, if $\frac{\beta_{1,1}}{\beta_{2,1}} < \frac{\beta_{1,2}}{\beta_{2,2}} < \frac{\beta_{1,3}}{\beta_{2,3}} < \dots < \frac{\beta_{1,T}}{\beta_{2,T}}$, there exists a time slot $t^*$ such that we have $P_{2,i} = 0$ for $\forall 1 \le i < t^*$ and $P_{1,i} = 0$ for $\forall t^* < i \le T$. In other words, there exists at most one (probably zero) time slot in which energy is generated by both power networks.

The proof is similar to that of Property 3 and thus it is omitted. In addition, assume that we have already known the value of $t^*$ based on the analysis in Section III.A. We also have the following property:

**Property 5:** At the optimal solution point of the above problem with $N = 2$ and $T > 2$, $\frac{\partial U/\partial P_{1,i}}{\beta_{1,i}}$ is the same for each $1 \le i < t^*$ and $\frac{\partial U/\partial P_{2,i}}{\beta_{2,i}}$ is the same for $t^* < i \le T$.

***Proof:*** Based on Property 4, we have $P_{2,i} = 0$ for $\forall 1 \le i < t^*$ and $P_{1,i} = 0$ for $\forall t^* < i \le T$, which means that $C_i = P_{1,i}$ for

$\forall 1 \le i < t^*$ and $C_i = P_{2,i}$ for $\forall t^* < i \le T$. Hence, Property 5 can be concluded from the proof of Property 1. ∎

Properties 4 and 5 can be used to determine the energy generation at all the other time slots when $P_{1,t^*}$ and $P_{2,t^*}$ are given:

$$P_{1,i} = \frac{\alpha_i}{\beta_{1,i} \cdot \sum_{m=1}^{t^*-1} \alpha_m} \cdot \left(I_1 - \beta_{1,t^*} P_{1,t^*}\right) \quad \forall\, 1 \le i < t^*. \tag{6}$$

$$P_{2,i} = \frac{\alpha_i}{\beta_{2,i} \cdot \sum_{m=t^*+1}^{T} \alpha_m} \left(I_2 - \beta_{2,t^*} P_{2,t^*}\right) \quad \forall\, t^* < i \le T. \tag{7}$$

As a result, given the value of $t^*$, there are only two variables in the above problem and they can be easily solved using geometric optimization. And also, we can use a similar procedure of determining the optimal value of $n^*$ discussed in the above subsection to determine the optimal value of $t^*$.

### *E. $N > 2, T > 2$:*

This is the most general scenario of the cooperative energy generation problem, which is based on the multi-country, multi-commodity model in international economics which has been studied in [10]. Although this is a complex problem with $N \cdot T$ variables and has been proved hard to solve, we can significantly reduce the problem complexity by combining the algorithms in the previous two subsections. Based on the property of energy generation cost, we are able to re-label all the power networks as well as the time slots so that for any $n_1 < n_2$ and $t_1 < t_2$, we have $\beta_{n_1,t_1}/\beta_{n_2,t_1} < \beta_{n_1,t_2}/\beta_{n_2,t_2}$. For convenience, we use the energy generation matrix **P** defined as:

$$\mathbf{P} = \begin{bmatrix} P_{1,1} & P_{1,2} & P_{1,3} & \cdots & P_{1,T} \\ P_{2,1} & P_{2,2} & P_{2,3} & \cdots & P_{2,T} \\ P_{3,1} & P_{3,2} & P_{3,3} & \cdots & P_{3,T} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{N,1} & P_{N,2} & P_{N,3} & \cdots & P_{N,T} \end{bmatrix}. \tag{8}$$

Based on the solution of the previous scenarios, at the optimal solution point, many of the items in the energy generation matrix will be zero. To study the solution of this general scenario, we also define a "path" of the energy generation matrix **P** in which the non-zero items form a path from the point (1,1) to (*N*,*T*). An example of a path for $N = 5$ and $T = 5$ is shown as follows:

$$\begin{bmatrix} P_{1,1} & P_{1,2} & 0 & 0 & 0 \\ 0 & P_{2,2} & P_{2,3} & P_{2,4} & 0 \\ 0 & 0 & 0 & P_{3,4} & 0 \\ 0 & 0 & 0 & P_{4,4} & 0 \\ 0 & 0 & 0 & P_{5,4} & P_{5,5} \end{bmatrix}. \tag{9}$$

**Property 6:** The energy generation matrix at the optimal solution point with $N > 2$ and $T > 2$ must be a path.

***Proof:*** If there is a non-zero item outside a path, we can find a pair of networks $n_1 < n_2$ together with a pair of time slots $t_1 < t_2$ that result in $P_{n_1,t_2} > 0$and $P_{n_2,t_1} > 0$. We can find a better solution according to the proof of Property 2. ∎

Dynamic programming algorithm can be used to examine all the paths in a matrix with a complexity of O((*N*+*T*)!). However, based on the optimal $n^*$ and $t^*$ determination methods in the previous subsections, we can define a "non-tortuous path" in which the inflection points are also set to zeros. This means on the non-tortuous path, we have either a power network is specified in energy generation at one certain time slot, or the energy at a time slot is generated by only one network, so it will be much easier to calculate the value of each item in the matrix. For example, a non-tortuous path of the corresponding path in (9) is given by :

$$\begin{bmatrix} P_{1,1} & P_{1,2} & 0 & 0 & 0 \\ 0 & 0 & P_{2,3} & 0 & 0 \\ 0 & 0 & 0 & P_{3,4} & 0 \\ 0 & 0 & 0 & P_{4,4} & 0 \\ 0 & 0 & 0 & 0 & P_{5,5} \end{bmatrix}. \tag{10}$$

Using the above definition, Algorithm 1 is presented to solve the cooperative energy generation problem with $N > 2$ and $T > 2$.

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**Algorithm 1: Solution for Cooperative Energy Generation Problem with $N > 2$ and $T > 2$.**

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**Initialize** $U = 0$:
// find optimal non-tortuous path
**For all** non-tortuous paths in **P**
    Calculate the value of each non-zero item in **P**;
    Calculate the utility function $U_{temp}$;
    **If** $U_{temp} > U$
        $U = U_{temp}$;
        Update the optimal non-tortuous path;
    **End if**
**End for**
// calculate optimal solution
**For all** paths corresponding to the optimal non-tortuous path
    Calculate the optimal value of each non-zero item in **P**;
    Calculate the utility function $U_{temp}$;
    **If** $U_{temp} > U$
        $U = U_{temp}$;
        Update the optimal energy generation solution;
    **End if**
**End for**

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Notice that different paths might share the same non-tortuous path, and also one path might correspond to more than one non-tortuous paths. The proposed algorithm has greatly reduced the complexity of the problem. The number of variables has been reduced from $(N \cdot T)$ to at most (*N*+*T*-1).

## IV. Experimental Results

To demonstrate the effectiveness of the proposed solutions, we examine various cases corresponding to the aforesaid (specific or general) scenarios. The proposed solutions have been implemented using C programming and tested for various cases. Because of space limitation, we only show the results for the special scenario $N = 2$, $T = 2$ and the general scenario $N > 2, T > 2$. The results from local optimization scenario ($N = 1$) are used as the baseline.

In the first experiment, we focus on the scenario $N = 2, T = 2$. The preference factors of the two time slots are set to be 0.3 and 0.7. Before trading, both networks are considered to be closed economic groups and maximize their own utility functions. When the two power networks open up to trade, they first get together to decide the optimal energy generation at each time slot so that the total utility can be maximized. After that, they distribute the total energy consumption in a fair way, as discussed in Section II.B. The model is tested for various cases with different combinations of energy generation cost and total resources. The detailed simulation results are presented in Table I.

TABLE I. SIMULATION RESULTS FOR DIFFERENT CASES WITH $N = 2$, $T = 2$

| | grid | $\beta_1$ | $\beta_2$ | $I_n$ | Before trade | | After trade | | $\frac{U_{trade}}{U_{local}}$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | $P_1$ | $P_2$ | $P_1$ | $P_2$ | |
| 1 | 1 | 1 | 2 | 10 | 3.0 | 3.5 | 3.0 | 3.5 | 1 |
| | 2 | 4 | 8 | 20 | 1.5 | 1.75 | 1.5 | 1.75 | 1 |
| 2 | 1 | 1 | 2 | 10 | 3.0 | 3.5 | 4.5 | 2.75 | 1.04 |
| | 2 | 6 | 8 | 20 | 1.0 | 1.75 | 0 | 2.5 | 1.04 |
| 3 | 1 | 1 | 2 | 10 | 3.0 | 3.5 | 10 | 0 | 1.08 |
| | 2 | 6 | 8 | 100 | 5.0 | 8.75 | 0 | 12.5 | 1.08 |
| 4 | 1 | 5 | 5 | 40 | 2.4 | 5.6 | 5.4 | 2.6 | 1.12 |
| | 2 | 8 | 4 | 40 | 1.5 | 7.0 | 0 | 10.0 | 1.12 |
| 5 | 1 | 1 | 5 | 50 | 15 | 7.0 | 30 | 4.0 | 1.33 |
| | 2 | 8 | 4 | 40 | 1.5 | 7.0 | 0 | 10.0 | 1.33 |
| 6 | 1 | 1 | 5 | 50 | 15 | 7.0 | 50 | 0 | 2.09 |
| | 2 | 8 | 1 | 40 | 1.5 | 28 | 0 | 40 | 2.09 |

One can observe from Table I that in case 1 with $\frac{\beta_{1,1}}{\beta_{2,1}} = \frac{\beta_{1,2}}{\beta_{2,2}}$, there is no comparative advantage between the two networks. As a result, the optimal solution can be achieved when each of the two networks simply maximizes its own utility, and none of the two networks can gain from trade. From case 2 to case 6, as long as there is a difference between $\frac{\beta_{1,1}}{\beta_{2,1}}$ and $\frac{\beta_{1,2}}{\beta_{2,2}}$, each power network can make use of its comparative advantage and achieves a utility increase from trading with each other. The higher this difference is, the more they can gain from trade. In addition, comparing case 2 and case 3, we can also observe that the total resource for energy generation will also affect the cooperative energy generation decision for both power networks. In case 2, the resource of the second power network is relatively limited, and thus the first network needs to generate energy at both time slots. But in case 3, both power networks have enough resources so that each of them turns out to generate energy at its own advantageous time slot.

In the second simulation, we analyze the general scenario with a total number of power networks $N = 4$ and total time slot $T = 6$. We set $\beta_{n_1,t_1}/\beta_{n_2,t_1} < \beta_{n_1,t_2}/\beta_{n_2,t_2}$ for any $n_1 < n_2$ and $t_1 < t_2$, and test our model in various cases with total resource combinations. The final results of cooperative energy generation at each time slot are shown in Table II.

TABLE II. COMPERATIVE ENERGY GENERATION RESULTS WITH $N = 4$, $T = 6$

| case | grid | $I_n$ | $P_{n,1}$ | $P_{n,2}$ | $P_{n,3}$ | $P_{n,4}$ | $P_{n,5}$ | $P_{n,6}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 900 | 225 | 113 | 56 | 28 | 0 | 0 |
| | 2 | 50 | 0 | 0 | 0 | 0 | 2.5 | 0 |
| | 3 | 50 | 0 | 0 | 0 | 0 | 5 | 0 |
| | 4 | 50 | 0 | 0 | 0 | 0 | 8.8 | 33 |
| 2 | 1 | 200 | 100 | 50 | 0 | 0 | 0 | 0 |
| | 2 | 200 | 0 | 0 | 16.7 | 0 | 0 | 0 |
| | 3 | 200 | 0 | 0 | 11.2 | 8.8 | 0 | 0 |
| | 4 | 300 | 0 | 0 | 0 | 19.2 | 55.8 | 112 |
| 3 | 1 | 30 | 26 | 2 | 0 | 0 | 0 | 0 |
| | 2 | 30 | 0 | 3.75 | 0 | 0 | 0 | 0 |
| | 3 | 30 | 0 | 3 | 0 | 0 | 0 | 0 |
| | 4 | 900 | 0 | 4.3 | 26 | 52 | 104 | 208 |

In the experiments shown in Table II, case 1 and case 3 are the two extreme cases in which one power network has much larger amount of resources and thus turns out to generate energy at most of the time slots, while the other networks generate energy at only one or two time slots. In case 2, as the four power networks have about the same amount of resources, the energy at each time slot turns out to be generated by one certain network with comparative advantage. No matter in which case, the optimal energy generation matrix turns out to be a path as we defined in Section III.E.

Another observation from the experimental results is that even if one power network has a much better technology, i.e., $\beta_{n,i}$ is smaller than any other networks at any time slot, it will still benefit from trading with other power networks. And also, a power network with less energy generation resources is more likely to turn out to generate energy at a single time slot.

## V. CONCLUSION

In this paper, we present an electricity trade model to deal with the cooperative energy generation and the energy distribution for multiple power networks in a smart energy system. In our model, each power network has its own energy generation and consumption and aims at maximizing its total user satisfaction level. We discuss the cooperative energy generation problem and solution in detail for different scenarios including the most general case where the proposed algorithm would significantly reduce the problem complexity while guaranteeing the optimality. The idea of comparative advantage is used for power networks in making decisions on energy generation. The accuracy and efficiency of our presented solutions are validated by experimental results.

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